Quantitative Agrobiology: III. The Mitscherlich Equation and Its Constants

O. W. Willcox

The criticisms aimed by Black, Kempthorne and White against Willcox's agrobiology in the current issue of Agronomy Journal are directed against three principal targets. First, they undertake to show that Willcox's derivation of the agrobiologic nitrogen constant 318 is mathematically specious, and that attribution of biological significance to the number 318 is therefore illusory. As if this were not enough to discredit the "so-called agrobiology", they go on with an effort to prove that there is no such thing as an inverse yield-nitrogen law.

In the preceding Part II hereof it was shown that Black and Kempthorne's case against the constant 318 is vitiated by their inattention to the mass action law, well known to chemists, physicists, and agrobiologists who have to deal with dynamic systems that approach equilibria, wherefore the Mitscherlich equation or its equivalent is applied to evaluate one variable in terms of an associated variable, both being dependent on the same independent variable. In Part I it was shown that their case against the inverse yield-nitrogen law is demolished by their own experiment that was expressly designed to test the validity of this law.

But even if their assaults on the constant 318 and on the inverse yield-nitrogen law were admitted to be failures, Black and Kempthorne still have a point which, if it could be made to stick, would ruin the foundation of quantitative agrobiology. This point is that the Mitscherlich equation itself is invalid and that its supposed constants are in fact variables. They open with the remark:

"Willcox's use of the Mitscherlich equation as the base for the 'derivation' of the constant 318 presupposes that the equation is suitable for the purpose. Willcox accepts the Mitscherlich equation as correct. In fact, he elevates it to the level of a 'Law'."

Here, undoubtedly, is the main issue of the whole dispute. Quantitative agrobiology has been put forward as a mathematical-biological discipline that embraces the whole world of rooted and green-leaved plants. If this discipline is found not to apply to all kinds of such plants and all essential factors of plant growth, it could hardly be called a science. It is therefore pertinent to determine whether the Mitscherlich equation is actually the authentic expression of a general law of the kingdom of plants.

LAW OF DIMINISHING INCREMENTS

To start on a search for a universal law of the plant world, one need look no farther than the law of diminishing increments of yield in agriculture. No matter how well any kind of plant is watered and fertilized, or how thoroughly it is cultivated, or how adequately it is protected from insects, disease or other physical injury, there is a point at which the supply of these factors is relatively small, the yield will begin tolarge; as the quantities of growth factors are increased there will also increase, but at a continually diminishing rate until finally the plants make no further response to increase of growth factors. This statement of the law of diminishing increments and of its mode of action is so thoroughly the experience of farmers through all the ages that it tries that no responsible authority in the upper ranks of agricultural instruction and research is likely to dispute it.

Origin of the Mitscherlich Equation

What has the Mitscherlich equation to do with the law of diminishing increments? This equation arose explicitly from a desire to find a mathematical expression for the law of diminishing increments. The deductive approach to such an equation is relatively simple. The known facts are: (1) in order to increase the quantity of growth factor is increased until a limit, designated as \( A \), is reached; and (2) throughout this process the increments of yield diminish smaller the nearer the obtained yield approaches the intermediate stage of the operation the actual yield, which will be smaller than \( A \); the difference between is, \( A - y \), represents the additional quantity of growth factor that must be obtained if the plants were supplied with additional yield reaches a limit-yield, designated as \( A \), will be smaller than \( A \); the difference between

Now \( (A-y) \cdot c \) is a function of the quantity or combination of growth factors, designated as \( c \) on the plants, so the situation at any stage is

\[
\frac{dy}{dx} = (A - y) \cdot c
\]

which on integration becomes

\[
\log (A - y) = \log A - c \cdot x
\]

or, if \( x \) is measured in baules:

\[
\log (A - y) = \log A - 0.301 \cdot x
\]

(The baule of a growth factor is that quantity necessary and sufficient to produce half of the normal yield \( A \). When \( x \) is measured in baules the first baule produces 50% of \( A \), two baules 75%, three baules 93.75%, and so on in the same diminishing order, until ten baules produce 99.9% of \( A \), which for purposes may be regarded as the limit of possibility. Such is the equation deduced by Mitscherlich from the experience of the mathematician Baule, for the ubiquitous law of diminishing increments of yield in agriculture. This equation is a test of the validity of this law of diminishing increments, the indisputable law of diminishing increments, and the existence of a definite limit of yield fixed by nature for every kind of plant; the approach to this limit by continual increments of yield; and, since the equation is derived from experiment, it would seem to have the validity of a law of nature.)