Competition for scarce water resources may lead to high water prices. Agricultural producers can adapt to increasing water price by adjusting irrigation management strategies. When water prices are high, deficit irrigation, the purposeful reduction of irrigation to reduce water use while accepting reduced yield, may increase net income. Deficit irrigation decisions depend on economic, biological, and physical conditions. A model based on relationships among the amount and cost of irrigation water applied, water productivity, and the value of water saved and crop yield was developed to assist farmers making deficit irrigation decisions. A model application for irrigated maize (Z. mays L.) production in northeast Colorado illustrates conditions under which deficit irrigation increases net income. High water costs and low crop prices favor deficit irrigation. When water has value for leasing, the combination of deficit irrigation production to reduce water use and water leasing may increase net income. Sufficiently high lease prices and low commodity prices favor rainfed production or fallowing.

Core Ideas

- Deficit irrigation may increase net farm income.
- Expensive water, low crop prices, and high elasticity of water productivity favor deficit irrigation.
- Combining economic with biophysical models improves applicability.
and English (1990) presented general relationships to evaluate the potential economic benefits of deficit irrigation when both production costs and crop yield are related to the amount of applied water. Models have been developed for Nebraska (Martin et al., 1989; Martin et al., 2010) and Kansas (Klocke et al., 2006) to allocate a limited irrigation water supply among several crops on a farm. Recent studies in Colorado analyzed the crop and water prices under which a farmer should deficit irrigate and lease a portion of his water supply for a given water production function (Manning et al., 2018; Varzi et al., 2019). Each of these models assumes fixed biophysical relationships among irrigation water applied, water consumed, and yield.

The objective of this paper is to present a model that farmers and water managers can use to evaluate the benefits of deficit irrigation. The model calculates the net income for an irrigated crop based on the amount and cost of irrigation, the amount and value of saved water, the amount and price of crop produced, and the costs of production. The model is based on customizable relationships among irrigation, water use, and yield, and provides useful insights for water managers in regions where water is scarce. For a set of water prices, production costs, irrigation and precipitation parameters, and water production function, the model indicates whether a farmer should fully irrigate, deficit irrigate, or not irrigate, and whether the farmer should purchase supplemental water or lease a portion or all of an available water supply to other users to maximize net income. The model applies most directly to irrigated farms in water-scarce regions but more generally provides a useful path for describing resource management decisions and their relationship to environmental and resource systems by embedding biophysical processes within an economic decision framework. We use the model here to demonstrate management options that maximize net income for irrigated maize (Z. mays L.) production in northern Colorado.

**MODEL DEVELOPMENT**

The model calculates net income per unit land area of a crop based on a quadratic crop water production function, amount of effective precipitation, irrigation efficiency, cost of the irrigation water supply, crop production costs, and revenue from selling the crop. The model includes potential income from deficit irrigating and leasing out the saved irrigation water. All biophysical variables are defined in terms of the relative evapotranspiration. Net income is maximized by optimizing the amount of water consumed by the crop.

**Water Production Function**

A water production function (WPF) defines the relationship between crop yield ($Y$) and crop water consumption (Barrett and Skogerboe, 1980; Doorenbos and Kassam, 1979; Geerts and Raes, 2009; Hanks, 1974; Steduto et al., 2007). Crop water consumption is the crop evapotranspiration (ET) that occurs in the process of producing the crop yield. Although only the transpiration component of ET contributes to yield production, both evaporation and transpiration contribute to water consumption and it is difficult to quantify evaporation separately from transpiration. Water production functions are determined by measurements of crop yield under varying levels of ET and vary among crops, the climate under which crops are grown, and management practices.

Crop potential ET ($ET_p$), the ET with no water deficit, varies with weather, so it is useful to normalize the WPF relative to $ET_p$. Crop potential yield ($Y_p$), the yield with no water deficit, varies with crop variety, management practices, and climate, so it is also useful to normalize the WPF relative to $Y_p$. Thus, a normalized WPF is relative yield ($Y/Y_p$) as a function of relative ET ($ET/ET_p$). This normalization is common in the literature (Doorenbos and Kassam, 1979; Geerts and Raes, 2009).

The shape of the WPF has been discussed in the literature. Many crop scientists have proposed that physiologically, biomass production is proportional to transpiration (Arkley, 1963; Briggs and Shantz, 1913; de Wit, 1958; Hanks, 1983; Sinclair et al., 1984; Steduto et al., 2007; Stewart et al., 1977; Tanner and Sinclair, 1983). However, field studies that include substantial deficits have found that the WPF relationship for marketable yield has a negative intercept; in other words, a quantity of ET is consumed before the first unit of marketable yield is produced (Doorenbos and Kassam, 1979; Hanks, 1983; Trout and DeJonge, 2017). This is expected since soil evaporation occurs during germination and early growth and some transpiration is required to create a plant capable of producing marketable yield. For grain crops, this minimum ET is typically 20 to 50% of $ET_p$. Many published studies propose that both biomass and grain yield increase linearly between this ET intercept and $ET_p$ (Cakir, 2004; Doorenbos and Kassam, 1979; Hanks, 1983; Stewart et al., 1974).

The impact of water stress on crop productivity varies with growth stage (Comas et al., 2019; Doorenbos and Kassam, 1979; Geerts and Raes, 2009; Hanks, 1983). Many annual seed-producing crops have varying responses to water stress during early plant formation, vegetative growth, flowering and seed formation, seed growth, and maturation stages. Thus, many crops have multivariate WPFs, and the final grain yield is a summation of the impacts of stress at each growth stage (Comas et al., 2019; Doorenbos and Kassam, 1979; Hanks, 1974). Conceptually, each crop has an optimum distribution of a seasonal water deficit across the season that minimizes yield loss. Strategically distributed seasonal deficits (i.e., deficits are applied first to the least sensitive growth stages) tend to result in curvilinear WPFs that are concave downward (negative second derivative) because deficits are initially applied at stages when the effect on yield is least, while severe deficits affect more...
yield-sensitive stages. Several studies with strategic water deficits have measured concave WPFs (Comas et al., 2019; Geerts and Raes, 2009; Manning et al., 2018; Trout and DeJonge, 2017). In this paper, a univariate crop WPF is used that represents a strategic within-season distribution of available water to the crop.

Based on past studies and these assumptions, a normalized quadratic WPF model is proposed. The independent variable is relative ET (ET/ETp), represented by x, and the dependent variable is the relative yield (Y/Yp), represented by YR:

$$Y_R = A + Bx + Cx^2$$  \[1\]

where A, B, and C are function coefficients. The end points of the normalized WPF are zero yield at the relative ET intercept (x0) and maximum relative yield (YR = 1) at maximum relative ET (x = 1). The quadratic relationship allows for both linear (C = 0) and curvilinear WPFs. If the relationship is curvilinear, it will generally be concave (C < 0) such that the rate of yield loss increases as ET decreases. The magnitude of the quadratic coefficient (C), defines the degree of curvilinearity in the WPF.

Figure 1 shows an example of this WPF model for x0 = 0.3 and C values of 0, -1, and -2. When C = 0, the relationship is linear. As C decreases (becomes more negative), the curvilinearity increases. Much of the curvilinearity is at high relative ET values, and the relationship approaches linearity as relative ET decreases.

When yield data are available at several levels of ET, the coefficients for Eq. [1] can be derived by regression analysis of the data, such as was done in Comas et al. (2019) and Trout and DeJonge (2017). If the ET data range is narrow and large deficits that produce low yields are not included, regression analysis may derive a linear WPF (nonsignificant quadratic term) with a positive intercept (A > 0). However, a positive intercept implies yield with no ET, which is not possible. Including yield data at low ET would likely result in a quadratic relationship with A < 0 and C < 0.

The A and B coefficients of Eq. [1] can also be derived algebraically if C and two points on the curve are known. Appendix 1 derives these coefficients based on the upper end point (x = 1, YR = 1) and yield at some other relative ET such as x = x0, and an assumed C value.

Irrigation Requirement and Supply Relationships

Irrigation requirement (Iq) is the amount of irrigation water that must be stored in the crop root zone to meet the target crop ET. It is the target ET minus precipitation and stored soil water that contributes to crop water use. The irrigation water applied (Iq) must be greater than Iq to account for irrigation water losses to deep percolation or runoff that do not contribute to ET. Although Iq, Iq, and precipitation provide water for ET and thus ET could be considered the dependent variable in the physical relationships, in this economic model, Iq and Iq are formulated as a function of relative ET so the costs associated with Iq and Iq can be combined mathematically with the income provided by yield via a single independent variable, x.

Not all precipitation is stored in the soil and used by the crop. Precipitation that exceeds soil infiltration capacity or water storage capacity runs off or deep percolates to groundwater. The effective in-season precipitation that contributes to ET (R) can be represented as the product of the precipitation amount (R [mm]) and the precipitation use efficiency (Er).

$$R = ErR$$  \[2\]

The predicted amount of in-season precipitation can be estimated from the historical average for the growing season (PRISM Climate Group, 2019). Ineffective precipitation can be quantified by runoff estimates and daily water balance calculations of deep percolation losses.

Precipitation contributes to crop ET indirectly between seasons by storing water in the soil for later use by a crop. When soil water storage is high throughout the season so that relative ET is high, there is little net use of stored preseason soil water and little capacity to store postseason precipitation. Effective storage (S) can be estimated as the product of the total plant-available soil water at the beginning of the season (S [mm]) and the net portion of S that is used by the crop (Es).

$$S = EsS$$  \[3\]

The total available soil water storage at the beginning of the season is limited by the total available soil water holding capacity or the effective off-season precipitation, and is reduced by off-season surface evaporation.

We model the efficiencies with which in-season precipitation and between-season soil water storage contribute to crop water use with a quadratic function of relative ET:

$$Er = \min\{D + Fx + Gx^2, 1\}$$  \[4\]

$$Es = \min\{H + Jx + Kx^2, 1\}$$  \[5\]

where D, F, G, H, J, and K are coefficients and Er and Es are limited to values less than or equal to 1.0. As with the WPF, the quadratic function coefficients can be derived from an assumed quadratic coefficient (G and K) that establishes the degree of curvilinearity and two endpoints of the relationship. Soil water deficits that cause low relative ET also result in high potential to store precipitation for crop ET. Irrigation management designed to meet full crop water requirements results in small soil water deficits and low potential to store precipitation. Thus, the portion of precipitation that contributes to crop water use tends to increase as the relative ET decreases. Thus, the two end points can be the relative ET below which all precipitation is used for ET (efficiency = 1.0) and the efficiency at ET (x = 1). With these end points, the coefficients are derived as:

$$F = \frac{Er_1 - 1 - G(1 - x_i^2)}{1 - x_i}$$  \[6\]

$$D = Er_1 - F - G$$  \[7\]

$$J = Es_1 - K(1 - x_i^2)$$  \[8\]

$$H = Es_1 - J - K$$  \[9\]

where x_i is the relative ET below which all in-season precipitation is effective, x_i is the relative ET below which all preseason available soil water storage is effective, Er_1 is the precipitation
use efficiency at $x = 1.0$, and $E_s_1$ is the soil storage use efficiency at $x = 1.0$.

The $x_r$ and $x_s$ end points will typically be between $0.3 < x < 0.6$, although economic model results are relatively insensitive to these parameters. $E_r_1$ will be large when $R$ is a small portion of full ET and will decrease with increasing precipitation amount and intensity as well as decreasing soil water storage capacity. In semiarid climates where $R$ is less than 50% of $ET_p$, $E_r_1$ will typically be larger than 0.5. Figure 2a shows an $E_r(x)$ relationship with $E_r_1 = 0.6, x_r = 0.3, \text{ and } G = -1$. $E_s_1$ will be small ($\leq 0.2$) when $x = 1$ because soil water storage would be near capacity at the end of the season. Figure 2b shows a typical $E_s(x)$ relationship with $E_s_1 = 0, x_s = 0.5, \text{ and } K = -2$. The irrigation requirement ($I_R [\text{m}^3 \text{ha}^{-1}]$) is the target ET not provided by $R$ and $S_e$:

$$I_R = 10[(ET_p - R_e(x) - S_e(x))]$$

where 10 is a unit conversion from mm to m$^3$ ha$^{-1}$. The irrigation requirement represents the amount of ET that is supplied by irrigation water. When $x$ is the relative ET under non-irrigated conditions, $I_R = 0$ and $ET_p = R_e(x) + S_e(x)$. $I_R$ is limited to non-negative values.

The irrigation water applied ($I_S [\text{m}^3 \text{ha}^{-1}]$) is the amount of irrigation water that must be applied to a crop to meet the target ET. $I_S$ is greater than $I_R$ due to irrigation inefficiency, or irrigation water lost to runoff and deep percolation due to non-uniform irrigation water distribution, non-uniform soil water storage capacity, and imperfect irrigation management.

$$I_S = I_R / E_i$$

As with precipitation efficiency, irrigation efficiency ($E_i$) also decreases as crop irrigation is increased to meet full crop water requirements (Heermann and Solomon, 2007; Sepaskhah and Ghahraman, 2004). $E_i$ can be modeled with the quadratic relationship:

$$E_i = \min\{[(L + Mx + Nx^2), 1]\}$$

where $L$, $M$, and $N$ are coefficients. As above, coefficients are determined by assuming $E_i = 1$ at $x \leq x_i$, $E_i = E_i_1$ at $x = 1$, and a degree of curvilinearity, $N$. Thus:

$$M = \frac{E_i_1 - 1 - N(1 - x_i^2)}{1 - x_i}$$

$$L = E_i_1 - M - N$$

$E_i_1$ depends on the irrigation system (application precision and uniformity) and management (irrigation scheduling and frequency) and will usually be between 0.5 (surface irrigation and poor management) and 0.9 (drip irrigation and excellent management) (Trout and Kincaid, 2007). Figure 2c shows an $E_i(x)$ relationship with $E_i_1 = 0.6, x_i = 0.3, \text{ and } N = -0.5$.

Relationships for precipitation, storage, and irrigation efficiency such as those in Figure 2 result in production function relationships between yield and $I_R$ or $I_S$ having lower slope and greater curvilinearity than the WPF based on ET. Even if the WPF based on ET is linear, the production function based on irrigation amount is curvilinear.

**ECONOMIC MODEL**

The economic model sums revenue from crop sales plus income from leasing a portion of the water supply and subtracts...
both fixed and variable costs of production to calculate the net income (NI). The model is developed for a single crop, and incomes and costs are calculated on a per unit area basis (ha\(^{-1}\)). All terms of the NI relationship are written in terms of relative ET so NI\(\left( x \right)\) can be calculated and optimized. The model is not designed to allocate water among different crops, although by determining NI\(\left( x \right)\) for several crops, a farmer can make informed water and land allocation decisions that maximize income, given a fixed land constraint and cropping objectives. Once the target relative ET is determined, the required amount of irrigation water to apply or lease can then be calculated.

The model assumes three production cost components. The first is the fixed costs of the farming operation, expressed on a per land unit basis, that do not depend on the target irrigation amount or yield. This includes the annualized cost of land and equipment and the base costs of supplies and labor to crop land for production. Annual land and equipment costs are assumed to be allocated evenly throughout the irrigated area in the farming operation. Fixed costs also include the base costs of the irrigation water supply, such as annualized costs of pumps and other irrigation equipment and fixed water supply payments such as the annual water share price paid to a water supplier.

The second production cost is the variable cost associated with producing an irrigated crop but not directly related to water use. This includes the costs of additional seed, fertilizer, pest control, and labor required to produce the yield associated with irrigated production. The variable costs are assumed proportional to the targeted yield, \(Y\left( x \right)\).

The third production cost is the variable cost of irrigation water beyond the fixed base costs. These may include: energy for pumping water, labor for irrigating, equipment maintenance associated with the volume of water applied, and any other variable costs associated with securing or applying irrigation water. Variable irrigation costs are assumed proportional to the volume of water applied, \(I_S\left( x \right)\).

The model includes two sources of income. The first is the revenue associated with the sale or value of the produced yield, which is calculated as the market price for the crop times the yield calculated from the WPF. The second is the potential income associated with leasing a portion of the water supply to others, which is calculated as the net lease price times the amount of water leased.

The leased water amount is the difference between the irrigation water right and the amount applied to the crop. The irrigation water right is a legal right, contract, allowance, or capacity to divert water from a surface supply, pump water from groundwater, or consume irrigation water. The basis for an irrigation water right is determined by water law in the country (in the United States, states legislate water law). We assume that the water right does not exceed either \(I_S\) or \(I_R\) for full irrigation, that is, the farmer cannot lease more water than is required to fully irrigate.

Based on these conditions, net income on a per unit land area basis is:

\[
NI\left( x \right) = PyY\left( x \right) + Pw\left[ Wc - I_S\left( x \right) \right] - Pps - PpY\left( x \right) - PiI_L\left( x \right)
\]

when the leasable amount of water is based on the irrigation water supply;

\[
NI\left( x \right) = PyY\left( x \right) - Pps - PpY\left( x \right) - PiI_L\left( x \right)
\]

when water leasing is not an option; where:

\[
NI\left( x \right) = \text{the net income from irrigated crop production and water leasing (}$ ha^{-1}\text{)}
\]

\[
P_Y = \text{the unit price of crop yield (}$ kg^{-1}\text{)}
\]

\[
P_W = \text{the unit price of leased irrigation water (}$ m^{-3}\text{)}
\]

\[
P_{ps} = \text{the fixed cost of production (}$ ha^{-1}\text{)}
\]

\[
P_v = \text{the variable cost of production, not including irrigation (}$ kg^{-1}\text{)}
\]

\[
P_l = \text{the variable cost of applied irrigation water (}$ m^{-3}\text{)}
\]

\[
Y\left( x \right) = \text{projected crop yield for the ET target (kg ha}^{-1}\text{)}
\]

\[
Wc = \text{the right based on irrigation water requirement (m}^3\text{ ha}^{-1}\text{)}
\]

\[
I_S\left( x \right) = \text{amount of irrigation water consumed (m}^3\text{ ha}^{-1}\text{)}
\]

\[
I_L\left( x \right) = \text{amount of irrigation water applied (m}^3\text{ ha}^{-1}\text{)}
\]

Note that Eq. [17] is a simplification of Eq. [15] and [16] with \(P_W = 0\). The three variables in Eq. [15–17], \(Y\left( x \right), I_R\left( x \right),\) and \(I_S\left( x \right)\), are functions of the relative ET. By substituting Eq. [1-3], [10], and [11] into Eq. [15–17], NI can be calculated with \(x\) as the independent variable.

\[
NI = PyY\left( x \right) \left[ A + Bx + Cx^2 \right] + Pw\left[ Wc - R\left( D + Fx + Gx^2 \right) - S\left( H + Jx + Kx^2 \right) \right] - Pps - PpY\left( x \right) \left[ A + Bx + Cx^2 \right] - PiI_L\left( x \right) \left[ E + Fx + Gx^2 \right] - PiI_L\left( x \right) \left[ H + Jx + Kx^2 \right]
\]

\[
NI\left( x \right) = \text{the net income from irrigated crop production and water leasing (}$ ha^{-1}\text{)}
\]

\[
I_S\left( x \right) = \text{amount of irrigation water consumed (m}^3\text{ ha}^{-1}\text{)}
\]

\[
I_L\left( x \right) = \text{amount of irrigation water applied (m}^3\text{ ha}^{-1}\text{)}
\]

\[
Py = \text{the unit price of crop yield (}$ kg^{-1}\text{)}
\]

\[
Pw = \text{the unit price of leased irrigation water (}$ m^{-3}\text{)}
\]

\[
Pps = \text{the fixed cost of production (}$ ha^{-1}\text{)}
\]

\[
Pp = \text{the variable cost of production, not including irrigation (}$ kg^{-1}\text{)}
\]

\[
P_l = \text{the variable cost of applied irrigation water (}$ m^{-3}\text{)}
\]

\[
Y\left( x \right) = \text{projected crop yield for the ET target (kg ha}^{-1}\text{)}
\]

\[
Wc = \text{the right based on irrigation water requirement (m}^3\text{ ha}^{-1}\text{)}
\]

\[
I_S\left( x \right) = \text{amount of irrigation water consumed (m}^3\text{ ha}^{-1}\text{)}
\]

\[
I_L\left( x \right) = \text{amount of irrigation water applied (m}^3\text{ ha}^{-1}\text{)}
\]
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Table 1. Coefficients and parameters for the model application.

<table>
<thead>
<tr>
<th>Parameter†</th>
<th>Value</th>
<th>Unit</th>
<th>Coefficient‡</th>
<th>Value</th>
<th>Price/cost§</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_p$</td>
<td>12,500</td>
<td>kg ha$^{-1}$</td>
<td>A</td>
<td>−2.07</td>
<td>$P_y$</td>
<td>0.18</td>
<td>$/kg$</td>
</tr>
<tr>
<td>$ET_p$</td>
<td>630</td>
<td>mm</td>
<td>B</td>
<td>5.98</td>
<td>$P_w$</td>
<td>0.20</td>
<td>$/m^3$</td>
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<tr>
<td>$x_0$</td>
<td>0.44</td>
<td>–</td>
<td>C</td>
<td>−2.91</td>
<td>$P_p$</td>
<td>0.05</td>
<td>$/kg$</td>
</tr>
<tr>
<td>$R$</td>
<td>215</td>
<td>mm</td>
<td>D</td>
<td>0.87</td>
<td>$P_{ps}$</td>
<td>1200</td>
<td>$/ha$</td>
</tr>
<tr>
<td>$ER_1$</td>
<td>0.6</td>
<td>–</td>
<td>F</td>
<td>0.73</td>
<td>$P_i$</td>
<td>0.04</td>
<td>$/m^3$</td>
</tr>
<tr>
<td>$x_r$</td>
<td>0.3</td>
<td>–</td>
<td>G</td>
<td>−1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>80</td>
<td>mm</td>
<td>H</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{s1}$</td>
<td>0.5</td>
<td>–</td>
<td>K</td>
<td>−2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{r1}$</td>
<td>0.6</td>
<td>–</td>
<td>L</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$x_{i1}$</td>
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<td>–</td>
<td>M</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$W_c$</td>
<td>5000</td>
<td>m$^3$ ha$^{-1}$</td>
<td>N</td>
<td>−0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

†Biophysical parameters
‡Function coefficients (input or derived from biophysical parameters)
§Economic parameters

Note that 1 ha-mm = 10 $m^3$ so that 630 mm = 6300 $m^3$ ha$^{-1}$

\begin{align*}
\text{NI} &= \text{Py} Y_p (A + B x + C x^2) \\
&+ \text{PW} \left[ W_s - ET - \left( D + F x + G x^2 \right) - S \left( H + F x + K x^2 \right) \right] \\
&- \text{PPS} \text{PP} Y_p (A + B x + C x^2) \\
&- \text{PI} \left[ ET - \left( D + F x + G x^2 \right) - S \left( H + F x + K x^2 \right) \right] \\
&= \text{Py} Y_p (A + B x + C x^2) - \text{PPS} \text{PP} Y_p (A + B x + C x^2) \\
&- \text{PI} \left[ ET - \left( D + F x + G x^2 \right) - S \left( H + F x + K x^2 \right) \right]
\end{align*}

Note that, as shown in Eq. [4], [5], and [12], if $x < x_o$, $x_r$, or $x_i$; $Er$, $Es$, or $Ei$, respectively, should be equal to 1.0.

Equations [15a–17a] can be optimized to find the relative ET that maximizes NI ($x_{op}$). An analytical solution was not found, so the solution must be calculated using numerical methods, which can be done through the “Solver” add-in routine for Microsoft Excel (Microsoft, 2018). The functions are concave and have only one global maximum, and $x_{op}$ must be between $x_0$ and 1. An approximate analytical solution to Eq. [15a] and [17a] is given in Appendix 2.

Possible solutions to the optimization include: full irrigation ($x_{op} = 1$), deficit irrigation to reduce irrigation costs or to allow water leasing ($R_e + Se) / ET_p < x_{op} < 1$), or no irrigation ($x_{op} \leq R_e + Se) / ET_p$ such that $I_e = I_e = 0$) with the land converted to rainfed cropping or fallow. The solution can be applied to all irrigable area up to the available water supply.

This biophysical and economic model of deficit irrigation is available in Excel spreadsheet format at the USDA Ag Data Commons site. DOI: 10.15482/USDA.ADC/1504421.

MODEL APPLICATION

Data from a 4-yr maize water productivity field study in northeastern Colorado were used to derive a water production function for maize (Trout and DeJonge, 2017). Mean yield and ET at full irrigation were 12,500 kg ha$^{-1}$ and 630 mm, respectively; with a relative ET intercept ($x_0$) of 0.44 and a curvilinear WPF with $C = 2.91$. In the region of this study at the western edge of the US Central Plains, the mean seasonal precipitation is 215 mm, and typical total available root-zone soil water storage at the beginning of the season is 80 mm. Table 1 lists these WPF, precipitation, and soil water storage parameters, plus the efficiency coefficients and parameters that were used in Figure 2.

In Colorado, a transferable water right is based on historic consumptive use which is the irrigation requirement ($I_R$) averaged over several years. It cannot exceed the irrigation requirement for full irrigation ($I_R$ for $x = 1$), but may be inadequate for full irrigation if the irrigated area has historically been deficit irrigated. Assuming the crops have historically been fully irrigated, the water right and maximum amount of the ET$^p$ that could be leased to other users is 5000 m$^3$ ha$^{-1}$ (Eq. [10] with $x = 1$).

Maize costs of production in the region include fixed costs of $1200 ha^{-1}$, variable production costs of $0.05 kg^{-1}$, and variable costs of irrigation of $0.04 m^{-3}$ (see Appendix 3 for the derivation of costs). Maize prices over the last 10 yr have varied widely between $0.12 kg^{-1}$ to $0.30 kg^{-1}$. Annual water lease prices in the region vary from $0.04 m^{-3}$, for leases between farmers or from municipalities that have excess water supplies to farmers, to $0.40 m^{-3}$, for lease agreements from farmers to water short municipalities, to as high as $1.0 m^{-3}$, to energy companies for developing oil wells (Varzi and Grigg, 2019a).

Figure 3 shows how NI per hectare varies with relative ET for a maize yield price of $0.18 kg^{-1}$ and a water lease price of $0 (no lease market, dashed line). For these production costs and this maize price, a grower makes a small profit (NI = $90 ha^{-1}$) with full irrigation ($x = 1$) and maximum yield. With the relatively low variable cost of irrigation water and no lease market, the optimum NI occurs at a relative ET of 0.93. The last 7% of ET to produce maximum yield requires 200 mm of irrigation water (2000 m$^3$ ha$^{-1}$) which costs $80 ha^{-1}$ compared to the $60 value of the additional 350 kg ha$^{-1}$ of yield produced. For these costs and crop price, the farmer loses money for $x < 0.78$. Even with reduced production costs, rainfed maize production is not viable in the region of the field study due to inadequate rainfall ($R_e + Se = x_0 ET_p$) resulting in very low average yields.

When the farmer has the option to lease a portion of his consumptively used irrigation water ($I_R$) for $0.20 m^{-3}$ (solid line in Fig. 3) the potential NI increases to $472 ha^{-1}$ at an optimum relative ET of 0.76. At this relative ET, although the income from yield would be $500 ha^{-1}$ less than at full yield, income from leased water is $525 ha^{-1}$ and production costs, including costs of...
irrigation, are reduced by $350 ha^{-1}$, resulting in the $375 ha^{-1}$ increase in NI. At larger deficits (lower relative ET), the loss of yield income is greater than the value of the additional leased water and the NI decreases and reaches $0 at x = 0.48. At a relative ET of 0.47, $I_R = 0$ (no irrigation) all the irrigation water is leased but the predicted rainfed yield is only 1000 kg ha^{-1}, resulting in a net loss of $60 ha^{-1}$. At $x_0 = 0.44$ no yield is produced so production costs exceed water lease income by $200 ha^{-1}$.

Since rainfed crop production is not economically viable in this low-rainfall semiarid climate, a farmer might choose to lease all his water right and fallow his land to minimize costs. Fallow land would incur costs to control weeds and erosion, but if fixed costs of land and equipment could be minimized, fallowing and water leasing can be profitable. For example, if fixed costs were reduced by 50% to $600 ha^{-1}$, and the annual land maintenance costs were $100 ha^{-1}$, total costs would be $700 ha^{-1}$ compared with potential lease income of $1000 ha^{-1}$ ($5000 m^3 ha^{-1}$ at $0.20 m^{-3}$) leaving NI of $300 ha^{-1}$.

Although this is $210 ha^{-1}$ more than for full irrigation, it is $170 ha^{-1}$ less than the maximum NI for deficit-irrigated maize. For these costs and the $0.18 kg^{-1}$ yield price, fallowing would provide the highest NI for lease water prices above $0.30 m^{-3}$. In regions where rainfall is sufficient to produce economically viable crops, the yield income from non-irrigated crop production would likely increase NI above that for fallowing.

Note on Figure 3 that the approximate solution, $x_{op}'$, (Eq. [A5]) is very close to the true optimum for $Pw = 0.2 m^{-3}$ but, when $Pw = 0 m^{-3}$, is higher than the true optimum by 0.03 and underestimates NI by $6 ha^{-1}$.

Figure 4 shows how the optimum relative ET (dashed lines, left axis) and maximum net income (solid lines, right axis) vary with crop price for three prices of irrigation water when leasing water is not an option. As expected, the optimum relative ET increases with crop price but decreases with increasing water cost. At each water price, maximum NI increases nearly linearly with crop price. The $0.04 m^{-3}$ variable irrigation cost is comparable to current water prices among farmers on a ditch system, prices a municipality might charge for their excess water supply, or the energy cost to pump shallow groundwater for a low-pressure center pivot irrigation system. At this relatively low water cost, the crop price must exceed $0.17 kg^{-1} for farmers to make a profit. If supplemental irrigation water can be purchased and applied for $0.04 m^{-3}$ or less, the farmer could purchase additional irrigation water to profitably irrigate additional land. As the expected crop price increases, the farmer should decrease the deficit (increase $x$) to maximize NI. If the crop price exceeds $0.22 kg^{-1}$, the farmer could increase farm income by purchasing additional water for up to $0.16 m^{-3}$ to irrigate additional land, but should increase the deficit to 20% to maximize profit with more expensive water.

Maize prices in the United States have been below $0.17 kg^{-1}$ since 2014. This means that even at low water prices, farmers have been losing money growing irrigated maize. The fact that farmers in the region continue to produce maize implies that either their costs of production are less than assumed in this application, or they are willing to absorb negative net annual income to continue the farming operation.

Figure 5 shows how the optimum relative ET and maximum NI vary with the lease price of water for $Pi = 0.04 m^{-3}$ and three maize prices. As expected, the optimum relative ET (dashed lines) is high for low lease prices and decreases as the water lease price increases and as the maize price decreases. High lease prices and low commodity prices favor deficit irrigation.

The solid lines in Figure 5 show that, as expected, the maximum NI increases with both the maize and water lease prices. For $Py = 0.15 kg^{-1}$, which represents the maize market price in the United States since 2014, the lease price must exceed $0.14 m^{-3}$ for positive NI. If water leasing is possible, a farmer can increase NI by increasing the deficit and leasing a portion of his water. For example, if the lease price is $0.20 m^{-3}$, the NI would increase from $372 ha^{-1}$ to $674 ha^{-1}$ by increasing the deficit from 7% ($x_{op}$ with $Pw = 0$) to 21% and leasing the saved water.

Also shown in Figure 5 (dotted line) is the NI that would result from leasing all the leaseable water, $Wc$, and reducing costs...
by fallowing the land, assuming fallowed land has annual costs of $700 ha$^{-1}$. At lease prices above $0.15$ m$^{-3}$, lease income would exceed these costs and would also exceed the maximum NI for a $0.15$ kg$^{-1}$ maize price. At a $0.36$ m$^{-3}$ lease price, income from leasing would exceed the maximum NI for a $0.20$ kg$^{-1}$ maize price. If costs can be reduced substantially, fallowing can provide higher NI than irrigation production when the crop price is low and water lease price is high. Where rainfed crop production is economically viable, the NI from a non-irrigated crop would likely exceed that for fallowing.

The results shown in Figures 4 and 5 were developed by numerical optimization of Eq. (15a) using the “Solver” add-in routine for Microsoft Excel (Microsoft, 2018). Using the generalized reduced gradient (GRG) nonlinear solving method, we maximized Eq. (15a) by choosing $x$ subject to the constraint that $x$ is between $x_0$ and 1.

For the parameters used in these figures, the approximate solution, $x_{op}$ (Eq. [A5]) overestimated $x_{op}$ by less than 0.06 when there was no leasing and less than 0.04 with leasing. Errors decreased with increasing water lease price and decreasing irrigation supply price because of the relative decline in importance of irrigation water supply cost (the last term in Eq. [15a]). With these approximation errors of $x_{op}$, NI was less than the true maximum NI by less than $10$ ha$^{-1}$ without water leasing when water supply costs were $0.04$ m$^{-3}$ and for all price combinations with leasing.

**DISCUSSION**

The presented biophysical and economic model calculates NI when irrigation water is applied to a crop or allocated between a crop and leasing. This approach facilitates analysis of the impact of the WPF, irrigation efficiency, precipitation, and costs and prices on decisions to deficit irrigate, purchase supplemental water, or lease irrigation water. By application of the model to several crops, the optimum relative ET and maximum NI can be compared among crops. Although allocating all water to the crop with the highest NI per unit area may maximize NI for the farming enterprise, management and resource priorities and constraints not included in the model will often motivate allocation of water and land among several crops. This simple model is valuable for guidance, but is not capable of water allocation optimization among crops of an irrigated farming enterprise. Also, by assigning a fixed production cost per hectare, the model abstracts away from the choice of the quantity of irrigated land. To the extent the farming enterprise has capacity to expand irrigated area, expanded area may reduce fixed production costs. Irrigation farmers often face limits on the amount of land they can or choose to irrigate. Likewise, reduction of irrigated area may increase fixed per hectare production costs.

This deficit irrigation economics model requires five price or cost parameters and 15 biophysical parameters. Many farmers can estimate their crop price, production, and water cost parameters. Enterprise cropping budgets prepared by university agricultural economists in many states can assist with cost and price estimates (e.g., CSU ABM, 2017). The model will be most useful if commodity and water markets are well established and relatively stable. Biophysical parameters are difficult for most farmers to estimate and guidance from university extension or USDA personnel will often be required. The most critical parameters are those used to calculate the WPF, especially the curvilinearity of the function, C. These can be estimated from field research data for a crop in a region. Alternatively, survey data of regional crop yields, such as those compiled by the USDA National Agricultural Statistics Service for many crops in the United States (USDA–NASS, 2019), can be combined with regional precipitation data (PRISM Climate Group, 2019) to estimate yield responses of nonirrigated crops to water availability. The model is relatively insensitive to the threshold relative ET values of the water relationships ($x_{r}, x_{s}, r, s$, and $x$) and only moderately sensitive to the remaining six water efficiency parameters if the cost of irrigation water is not high.

The relative ET that provides the maximum NI decreases (greater deficit) with crop price and when water cost and lease price increase. The irrigation water cost increases not only with the price, but also with reduced irrigation efficiency or precipitation. Full irrigation (no water leased) maximizes NI when the crop price is high and the irrigation water cost and lease price are low. No irrigation (all water leased) maximizes NI when the crop price is low and the irrigation water cost or lease price is high. The range of prices over which deficit irrigation is economically desirable is constrained by these upper and lower bounds (Manning et al., 2018; Varzi et al., 2019).

Many deficit irrigation studies in the irrigation management literature propose that the goal under limited water conditions is to maximize water productivity (WP, yield per unit water used) (Bluemling et al., 2007; Pereira et al., 2012). This approach assumes the only production cost is the variable cost for water. For the WPF used in the model application, the relative ET that maximizes WP is 0.84. Figures 4 and 5 show that $x_{op}$ for this WPF varies widely depending on prices. Although NI may be related to WP (Rudnick et al., 2016), selecting $x$ to maximize WP does not maximize NI for most conditions, and should not be used to select deficit irrigation management strategies.

The relationships in Figures 3 to 5 are based not only on the yield and lease price, but also on variable production costs that decline as the planned yield declines. This requires that the farmer can predict prices, correctly select the target relative ET and yield, and apply production inputs (primarily seed and fertilizer) appropriate for the target yield. Once a crop is planted, many of the variable production costs are spent and the ability to appropriately adjust production inputs and costs, other than variable irrigation costs, is limited. If prices change such that $x_{op}$ and target yield...
decreases, it may be impossible for the farmer to reduce production input costs appropriate for the anticipated yield. If prices change such that $x_{\text{op}}$ and the target yield increases, it may be difficult for the farmer to increase inputs to achieve target yields. Thus, maximizing NI requires correctly anticipating prices. If the opportunity to lease water occurs after planting decisions are made and cropping costs expended, the lease price would need to be high enough to compensate for higher than required production costs under the reduced water supply condition.

If the farmer knows pre-season that the water supply will be inadequate to achieve $x_{\text{op}}$, the farmer should reduce irrigated area to reduce production costs and achieve $x_{\text{op}}$ on the remaining irrigated land. After variable production costs are expended, if the irrigation water supply is less than had been anticipated at planting but sufficient to produce yield on all planted land ($x > x_{\text{op}}$), the highest net income for a crop with a concave WPF would result from spreading the remaining water supply evenly across all cropped land.

The curvilinearity of a WPF strongly influences the benefits of deficit irrigation. For a given set of prices, a less curvilinear WPF (less negative C) results in higher $x_{\text{op}}$ (less deficit), lower predicted relative yield, and lower maximum NI. For the model application with no leasing, if $C = 0$ (linear WPF), $x_{\text{op}} = 1.0$ for all price combinations, and there is no economic benefit of deficit irrigation. Without leasing and $P_{\text{f}}$ = $0.04$ $\text{m}^3\text{ha}^{-1}$, full irrigation provides the maximum NI for all $C$ higher than $-2.0$. The WPF must have substantial curvilinearity for deficit irrigation to increase NI when there is no water leasing and the irrigation water price is low. If there is a lease value for the irrigation water, the response is complex and deficit irrigation may maximize NI even if the WPF is linear.

In the model application, leasable water is based on reductions in ET (consumed irrigation water) relative to a benchmark consumption, $W_c$. In Colorado, this benchmark is based on historic consumptive use of a farm (Varzi and Grigg, 2019a). Water accounting is based on ET because the water supply is fully allocated, and downstream surface, groundwater supplies, and water rights are dependent on historic return flows (deep percolation and/or field runoff) from upstream users. In watersheds where return flows are effectively reused within the watershed, water consumption determines the loss of water to the watershed (Clemmens et al., 2008; Grafton et al., 2018).

If water laws in a region allow leasing on the basis of irrigation water supply rather than water consumption, a larger volume of water can be leased for a given yield decline. For example, for the parameters used in the application (Table 1), a 20% yield loss would result from an ET reduction of 24% or 1500 $\text{m}^3\text{ha}^{-1}$, which would result from an irrigation requirement reduction of 2500 $\text{m}^3\text{ha}^{-1}$ (due to more effective use of precipitation and soil water storage), and an irrigation supply reduction of 5200 $\text{m}^3\text{ha}^{-1}$ (due to higher irrigation efficiency). For these conditions, the same NI would be generated with an irrigation water supply lease price about 50% lower than a consumptive use-based lease price.

Quantification of ET reductions with deficit irrigation is difficult, because measuring ET or estimating irrigation efficiency is difficult. Thus, there are costs associated with quantification and documentation of ET reductions for water leasing with deficit irrigation (Varzi and Grigg, 2019b). There may also be costs associated with transporting the water to the lessee and storing it until needed. Although these transaction costs are normally borne by the lessee, they may be large and will influence the lease price of water. The transaction costs may be avoided if the leased water is for similar uses in the same watershed (e.g., leased to nearby farmers).

In Colorado, water leasing agreements may be based on a long-term contract (often 10 yr or more) that pays a fixed annual amount for the right to lease water, when needed, at a fixed lease price (Environmental Defense Fund, 2016; Lorenz and Doherty, 2018; Varzi and Grigg, 2019a, 2019b). Thus, the lease value is a combination of a fixed annual contract payment and the water lease price. The contract payment can be used to offset fixed costs of production which would increase NI but not change $x_{\text{op}}$. The contract payment could instead be used to upgrade the irrigation system to reduce the variable costs of irrigation (e.g., energy or labor) which would increase both $x_{\text{op}}$ and NI. Irrigation system improvements that increase irrigation efficiency reduce the amount of irrigation water required and also increase both $x_{\text{op}}$ and NI.

Risks associated with deficit irrigation are not accounted for in this analysis. For concave WPFs, the sensitivity of yield to ET (marginal response) increases with deficit irrigation, implying that spatial yield variation due to irrigation or soil non-uniformity, and temporal variability due to precipitation uncertainty is greater with deficit than with full irrigation. However, with deficit irrigation, the crop is better able to use precipitation that exceeds expectations, as is quantified in the effective precipitation relationship (Eq. [2] and [3]). The model can help farmers assess risk by estimating yield and revenue responses to variations in relative ET. A risk that is difficult to quantify is yield loss that may also result from greater sensitivity of stressed crops to pest pressures. An additional risk consideration for irrigation farmers in the United States is that federal crop insurance programs do not currently insure deficit irrigated crops, due to the difficulty in estimating expected yields. Farmers should consider increased risk when making deficit irrigation decisions (Varzi and Grigg, 2019a, 2019b).

Water leasing to non-agricultural users reduces crop production, however, water leasing that increases farm NI may improve the sustainability of the farming operation (Best, 2017; Varzi and Grigg, 2019a). Leasing also enables production sustainability by leaving the water resource under the control of the farming operation, as compared to water rights purchases that usually transfer water resource control to a non-agricultural buyer. Thus, although water leasing reduces crop production in the short term, to the extent that the increased NI is reinvested in the farming operation and control of the water resource remains in the agriculture sector, it likely improves the sustainability of production in the long term.

**CONCLUSIONS**

Net income from a combination of irrigated crop production and water leasing was calculated based on a model that describes the biophysical system, costs of production, and the value of crop yield and leased water. The NI model written in terms of crop ET was optimized to maximize NI from a unit of land. The optimal solution depends on costs and prices as well as the WPF. The model determines whether farmers should fully irrigate a crop, deficit irrigate to reduce water costs or generate income from leasing water, or not irrigate and lease the total water supply. High water cost and lease price, low crop price, and curvilinear WPFs favor deficit irrigation or no irrigation. Low water cost or value favor full irrigation. The model can assist farmers to allocate scarce water supplies.
APPENDIX 1: PARAMETERIZATION OF THE WATER PRODUCTION FUNCTION

The $A$ and $B$ coefficients of Eq. [1] can be derived algebraically if $C$ and two points on the curve are known. Because the WPF is normalized, $Y_R = 1$ at $x = 1$ is always the upper end point on the curve. If the relative yield, $Y_{R2}$, is known at some other relative ET, $x_2$, the coefficients can be derived:

$$A = Y_{R1} - x_1 + Cx_1^2 + Cx_2^2 - 1$$

$$B = Y_{R1} - x_1 - Cx_1 + Cx_2 - 1$$

This second point in semiarid regions might be the relative yield under unirrigated (rainfed) conditions if the effective rainfall that contributes to crop growth is assumed equal to ET. When the amount of ET required to initiate marketable yield, $x_0$, is known or can be estimated, the second point could be $Y_{R2} = 0$ at $x_2 = x_0$ and Eq. [A1] and [A2] simplify to:

$$A = \frac{C x_0 - x_0 - C x_1^2}{1-x_1}$$

$$B = \frac{1-C + C x_1^2}{1-x_1}$$

Note that for large values of $x_0$ and $C$, the WPF can project relative yields greater than 1. Although this is physically feasible, this condition should be avoided for most crops. To avoid this condition, the derivative of Eq. [1] at $x = 1$ must be greater than or equal to 0.

$$\frac{dY}{dx} = B + 2C \geq 0$$

$$C \geq B/2$$

APPENDIX 2: APPROXIMATE ANALYTICAL SOLUTION, $x_{op'}$

In most cases, the last term of Eq. [15a–17a] (variable cost of irrigation) is small relative to the remaining terms, and $NI(x)$ can be approximated by replacing $Ei$, the denominator in the last term, with a constant. This substitution enables an approximate analytical solution for Eq. [15a] and [17a] (but not [16a] which also contains $Ei$ in the second term). Making this substitution into Eq. [15a] and separating the constant, linear, and quadratic terms gives:

$$NI(x) = \frac{PyY_C + PwWS - PwFR + PwSJ}{-PPY_A - PPS + PIR}$$

$$+ \frac{PyY_B - PwET + PwRF + PwSJ}{-PPY_B - PPS + PIR}$$

$$+ \frac{PyYpC + PwRF + PwSK}{-PPYpC + PIR}$$

$$+ \frac{PyYpC + PwRF + PwSK}{-PPYpC + PIR}$$

The maximum NI is at the relative ET where the derivative $d(NI)/dx = 0$. Equating the derivative of Eq. [A4] to 0 and solving for $x$ gives the approximate optimum relative ET ($x_{op'}$):

$$x_{op'} = \frac{\left[PyY_C + PwRF + PwSJ + PwRF + PwSJ\right]}{-PPY_B - PPS + PIR}$$

The approximate solution for Eq. [17a] is the same as Eq. [A5] with $Pw = 0$. The approximate solution can be derived by estimating $Ei$, calculating $x_{op'}$, calculating $Ei$ at $x_{op'}$, substituting this value into Eq. [A5], and recalculating $x_{op'}$. With an initial $Ei$ estimate of 0.75, two $Ei$ iterations produce $x_{op'}$ within 1% of the true value. However, due to bias created by replacing $Ei$ by a constant, Eq. [A5] overestimates the true optimum and underestimates NI by a small amount, especially when $Pi$ is large or $Pw$ is small. This approximate optimum can be used as the upper limit for optimization of Eq. [15a] and [17a].

APPENDIX 3: EXPLANATION OF PRICES USED IN THE APPLICATION

Production costs were estimated from the 2016 Colorado State University Crop Enterprise Budget for irrigated and dryland corn in northeast Colorado (CSU ABM, 2017). Total operating costs for sprinkler-irrigated corn were estimated as $1571 ha$⁻¹, of which $556 ha$⁻¹ was for seed and fertilizer, and $515 ha$⁻¹ was for irrigation expenses. Property and ownership costs were $193 ha$⁻¹, and factor payments for land was $494 ha$⁻¹. Total operating costs for dryland maize production with a $5400 kg ha$⁻¹ yield was $506 ha$⁻¹, or one-third that for irrigated corn, or about 50% of the operational costs minus irrigation costs for irrigated maize. Irrigation costs for furrow-irrigated maize from surface water supplies were estimated as $142 ha$⁻¹. In this model application, we assume variable irrigation costs were $250 ha$⁻¹ for full irrigation ($6300 m^3 ha$⁻¹) or $0.04 m^3$, and the variable costs of production were $625 ha$⁻¹ to produce 12,500 kg of maize or $0.05 kg^{-1}$. This leaves $1200 ha$⁻¹ of sunk costs associated with the decision to plant maize on irrigated land.

APPENDIX 4: NOTATION

The following symbols are used in this paper:

$A$ = water production function intercept coefficient

$B$ = water production function linear coefficient

$C$ = water production function quadratic coefficient

$D$ = intercept coefficient for effective precipitation, $R_p$

$Ei$ = irrigation efficiency, portion of irrigation water applied that is used by the crop

$E_i$ = irrigation efficiency when ET = ET (x = 1)

$E_{irr}$ = effective precipitation when ET = ET (x = 1)

$E_s$ = portion of soil water storage that is used by the crop ET

$E_s$ = soil water storage that is used by the crop ET when ET = ET (x = 1)

$ET$ = evapotranspiration (mm)

$ET_p$ = evaportranspiration for a crop without water stress (mm)

$F$ = linear coefficient for effective precipitation, $R_p$

$G$ = quadratic coefficient for effective precipitation, $R_p$
\[ H \] = intercept coefficient for effective storage, \( \text{Es} \)
\[ I_R = \text{irrigation water that must be applied to meet the target irrigation ET} \ (\text{m}^3/\text{ha}^-1) \]
\[ I_S = \text{irrigation water that must be applied to meet the target} \ I_R \ (\text{m}^3/\text{ha}^-1) \]
\[ J = \text{linear coefficient for effective storage, Es} \]
\[ K = \text{quadratic coefficient for effective storage, Es} \]
\[ L = \text{intercept coefficient for irrigation efficiency, Ei} \]
\[ M = \text{linear coefficient for irrigation efficiency, Ei} \]
\[ N = \text{quadratic coefficient for irrigation efficiency, Ei} \]
\[ Pw = \text{the price of leased irrigation water} \ ($/\text{m}^-3) \]
\[ Pp = \text{the variable cost of production, not including irrigation} \ ($/\text{kg}^-1) \]
\[ Pps = \text{the fixed cost of production} \ ($/\text{ha}^-1) \]
\[ Wc = \text{water right based on historic consumption of irrigation water} \ ($/\text{ha}^-1) \]
\[ WS = \text{water right based on allowed surface water diversions or groundwater pumping} \ ($/\text{ha}^-1) \]
\[ WP = \text{normalized water productivity} = \ Y / x \]
\[ x_2 = \text{relative ET for which relative yield} = Y_{R2} \]
\[ x_i = \text{relative ET below which Ei} = 1 \]
\[ x_0 = \text{relative ET at which predicted yield is 0} \]
\[ x_p = \text{relative ET that produces the maximum net income} \]
\[ x_{op} = \text{the approximate relative ET that produces the maximum net income} \ (\text{Eq. [A5]} \]
\[ x_e = \text{relative ET below which Er} = 1 \]
\[ x_i = \text{relative ET below which Es} = 1 \]
\[ Y = \text{yield} \ (\text{kg/ha}^-1) \]
\[ Y_R = \text{relative yield} = Y / y \]
\[ Y_{R2} = \text{relative yield at} x = x_2 \]
\[ Y_p = \text{yield with no water stress} \ (x = 1) \ (\text{kg/ha}^-1) \]

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