$B^{-1} \theta_o \exp B \theta_o + B^{-2} (1 - \exp B \theta_o) = A^{-1} P T$. \[5\]

This last expression shows in particular that $\theta_o \to 0$ as $T \to 0$, as it must.

Consider now the case when $I > K(1)$ so that gravity is negligible up to the time of ponding $T_p$. The latter is given by Eq. [5] as the time for $\theta_o = 1$. Since the similarity solution breaks down as $T \to 0$, we want to determine the time above which it can be used, at least approximately. The similarity solution holds as long as $\theta_o (T)$ given by Eq. [5] is consistent with the similarity solution

$$\exp B (\theta_o - 1) = T/T_p.$$ \[6\]

This is clearly the case if $\theta_o$ is close to one, i.e. as long as $B^{-1} \ln T_p/T$ is small compared to one. For instance if $B = 8$ we expect the similarity solution to apply with better than a 10% accuracy when $T > T_p/2$. The similarity solution becomes increasingly worse for shorter times essentially because Eq. [16] of Ahuja and Römkens does not hold accurately and introduces a singularity in their solution.

In conclusion Ahuja and Römkens similarity solution is valuable and describes water infiltration into a soil accurately for a finite time interval. The upper bound of that interval can be as high as the ponding time if the imposed flux is sufficiently larger than the conductivity at saturation. The lower bound of the interval can be as low as half the ponding time and their solution still be reasonably precise.

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Reply to Dr. Parlane’s Letter

There is no doubt that the effect of gravity on flow during early stages would be negligible only as long as the rainfall intensity $I$ is much larger than the hydraulic conductivity $K(\theta_o)$, which changes with time as the water content $\theta_o$ at the soil surface $Z = 0$ changes. For the effect of gravity to be negligible up to ponding, $I$ must be larger than the conductivity at saturation. However, it is important to point out that the hydraulic conductivity decreases very sharply when the soil-water content is even slightly below the saturated value. This fact makes it easier for the condition of $I > K(\theta_o)$ to hold during early stages of infiltration of even a medium-intensity rainfall, where there would be an appreciable time involved before the water content at the soil surface reaches the saturated value. Of course, for rainfall intensities smaller than the saturated conductivity, the water content $\theta_o$ would never reach the saturated value, and the condition of $I > K(\theta_o)$ would be satisfied for a much longer time. In fact, we made a point in our paper that the similarity equations would be especially useful for describing early stages of infiltration of a medium- to low-intensity rainfall. We tested the applicability of similarity equation for wetting-front on all the available experimental data for medium- to low-intensities with good results. In these tests, we specifically verified the assumption of negligible gravity effect by applying the linear transformations [Eq. [5] of Ahuja and Römkens (1974)] to data for two or more different rainfall rates in a given soil. The results indicated an adequate validity of the transformations and hence the above assumption.

Dr. Parlane’s second point concerns the applicability of similarity equations near time $T = 0$. Our solution did show that water content $\theta_o$ at the soil surface would increase like $1/NT$, which has a singularity at $T = 0$. This expression for $\theta_o$ was in fact derived from the similarity solution for the soil-water diffusivity $D(\theta_o)$, which showed a proportional relationship between $D(\theta_o)$ and $T$. The similarity solution as a whole was based on the requirement that the initial (for $T = 0$) soil water content $\theta_o$ was low enough such that $D(\theta_o)$ was very small and effectively equal to zero during the flow. Since the water content on the soil surface would rise very rapidly just in a very small time to a level much higher than the initial water content, and consequently the soil-water diffusivity $D(\theta_o)$ would have even a more rapid rise, we feel that the above assumption of $D(\theta_o)$ being effectively equal to zero would hold after a very small time from the start of rainfall. Based on this, Dr. Parlane’s estimate of the lower limit of applicability of the similarity equations being equal to half the time for ponding appears to be rather high. Our feeling is supported by the experimental verifications that we presented or discussed in our paper, which showed that the similarity-based equation for the wetting-front adequately described the data including the earliest-recorded points close to $T = 0$ (also see Collis-George and Laryea, 1970).

The conclusion of Dr. Parlane may possibly be due to the fact that his Eq. [4] and [5] as given in his letter are only approximate and may not be so good especially during early stages of rain infiltration. These equations were derived with the assumption that the term $\partial Z/\partial T$ of the partial differential equation of flow is negligible or, in other words, the water flux is independent of position $Z$ (Parlange, 1972). This assumption, which is similar to the assumption in the well-known approach of Green and Ampt (1911), may not hold especially in soils (except, perhaps, a sandy soil) during the very early stages of rain infiltration, when the soil water content would continue to rise appreciably both at and near $Z = 0$.

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