Determination of Soil Water Diffusivity by Sorptivity Measurements

A recent paper by Dirksen (1975) discusses a method for the measurement of soil water diffusivity. The method assumes that the sorptivity, $S$, is related to a weighted mean diffusivity by the equation

$$\int_{\theta_o}^{\theta_1} (\theta - \theta_o)^\gamma D(\theta) d\theta = (1 + \gamma)^{-1} (\theta_1 - \theta_o)^{\gamma - 1} \Pi S^2/4. \tag{1}$$

The form of this equation is such that it becomes exact if the diffusivity $D$ is a constant. $\theta_o$ is the initial water content, $\theta_1$ the water content at the soil surface and $\gamma$ is a constant. Following a numerical test, Dirksen took $\gamma = 0.67$. Considering the usual scatter of absorption experiments, there is no question that Eq. [1] with $\gamma = 0.67$ is quite accurate for most practical purposes. However, rather than choose $\gamma$ by a numerical test it would be more satisfying from a theoretical point of view to take $\gamma$ so that Eq. [1] holds when $D$ approaches a delta function as well, or

$$\gamma = \Pi/2 - 1 = 0.57. \tag{2}$$

This is close enough to the value chosen by Dirksen to be just as accurate in any practical case.

Actually since $D$ varies very rapidly with $\theta$ for all soils, it is worthwhile to look for a relationship between the weighted diffusivity which becomes exact when $D$ approaches a delta function, i.e.,

$$\int_{\theta_o}^{\theta_1} (\theta - \theta_o)^\gamma D(\theta) d\theta = (\theta_1 - \theta_o).$$

Of course when $\gamma = 0.57$ Eq. [1] and [3] one does not require that Eq. [3] should hold for all cases then $\gamma$ is arbitrary. For instance, when Dirksen showed with his numerical example that a value of $D$ which is too large by 40% while a value which is too small but only by 10% points out the validity of Eq. [3] for real soils.

The best choice for $\gamma$ is now obtained by optimization. It can be shown (Parlange, 1975) that if $D$ varies near $\theta_1$ then

$$S^2 \approx \int_{\theta_o}^{\theta_1} (\theta - \theta_o + \theta_1 - \theta_o) D(\theta).$$

Expanding $(\theta - \theta_o)^\gamma$ in Eq. [3] for $\theta$ in the neighborhood of $\theta_1$ shows that Eq. [3] and [4] are identical to the first two orders when $\gamma = \frac{1}{2}$, or

$$S^2 = 2 (\theta_1 - \theta_o)^{\gamma/2} \int_{\theta_o}^{\theta_1} (\theta - \theta_o)^{\gamma/2} D(\theta).$$