those in Eq. [2]. This jump can be identified in the approaches of both groups of authors. [See, for example, the paragraph before Eq. [5a] in Bhattacharya et al. (1976c), or the section “Interpretation of the partial derivative $\partial D/\partial x$” in the paper by Laroussi and de Backer (1975).]

We have two objections to the approach. Firstly, we believe that it is physically unrealistic to attempt to develop a general water flow equation in which the flux is identified with a water content gradient. There are many physical situations where this has been experimentally demonstrated not to be the case, and for this reason such an approach must be rejected. Secondly, the argument in the “jumps” identified above is not mathematically justified. That of Bhattacharya et al., for example, effectively states that since $\theta$ may be a function of $x$ and $t$, it follows that $x$ and $t$ are defined by $\theta$, whence $A(x,t)$ can be equated with $D(\theta)$. Indeed, there is no more reason for making this assertion than for claiming that $D$ is a function of some differential of $\theta$ with respect to $x$ or $t$, except that the authors already know the form of the physically-based Eq. [2] they wish to “derive”. The “derivation” is then used to justify the Markovian assumptions. What is disturbing then is that the authors seek to introduce Markovian concepts into the physical “picture” of water movement. We believe that these ideas contribute nothing to our understanding of the physics of water movement in soils and are, indeed, needlessly confusing.

Furthermore, they contribute nothing to the understanding of situations where water movement does not occur in conjunction with a water content gradient; i.e., this theory appears to have nothing to say about redistribution, or any process in which hysteresis is significant.

Received 26 Jan. 1977.

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1. Structure of the Flux Law Based on the Markovian Postulate

Our Markovian postulate leads to the following special case of the volumetric flux density (Bhattacharya et al., Eq. [24]).

$$\mathbf{J} = -\nabla F_s(\theta, t, x) - \mathbf{a}(\theta, t, x)$$

According to Eq. [1], aside from the second component, the flux is equal to the gradient of a function of some differential of $\theta$, dependent on $x,t$, as well as on the initial moisture content of the system. We emphasize that, in Eq. [1], $\theta$ is a real valued variable, and Eq. [1] is applicable to any heterogeneous or homogeneous medium in which the flow is isothermal and isohaline, and the flow is in equilibrium with the atmosphere. We express that, for example, $\mathbf{J}$ is proportional to $\nabla \theta(t,x)$. If one assumes a homogenous medium, then $D_s(\theta, t, x) = D_s(\theta)$; $\mathbf{a}(\theta, t, x) = \mathbf{a}(\theta)$, and

$$\mathbf{J} = - \left( \frac{d}{d\theta} F_s(\theta) \right) \nabla \theta(t, x) - \mathbf{a}(\theta)$$

Equation [2], in which $\mathbf{J}$ is proportional to $\nabla \theta$, is a special case of Eq. [1]. Knight and Smiles claim that our theory only produces Eq. [2]. Nowhere do we make the naive and incorrect statement that “$x$ and $t$ are defined by $\theta$.” In a heterogeneous medium, $D_s(\theta, t, x) = D_s(\theta)$; $\mathbf{a}(\theta, t, x) = \mathbf{a}(\theta)$, and Eq. [1] will hold, but Eq. [2] cannot in general. Let us mention also that our work, at least, explains nor contradicts the phenomenon of hysteresis in soils. It is in consonance with the Buckingham-Darcy law itself.

2. Linearity Versus Quasi-Linearity: The Mathematical Issue

The commonly used parabolic Eq. [2] in Knight and Smiles (1977) is quasi-linear in the sense that the coefficients of the dependent variable $\theta$. The transition density of the Dirichlet process, on the other hand, satisfies a linear equation (Gikhman and Skorokhod, 1969, p. 377) which, in one dimension, is given by:

$$\frac{\partial p}{\partial t} = \frac{\partial^2}{\partial x^2} \left[ D_s(t, x) p \right] - \frac{\partial}{\partial x} [a(t, x) p]$$

where $p(0,x_0,t,x)dx$ is the probability that a Markovian particle in the infinitesimal interval $[x,x+dx]$ at time $t$ was at $x_0$. In our work, the coefficients of the flux equation (namely, a water quasi-molecule or darcion) were assumed to be constant, and Eq. [1] was derived from that.

[The remainder of the text is not included due to the length of the document.]