References


Comments on “Steady Infiltration from Spherical Cavities”

In a recent paper, Philip (1984) discusses the steady infiltration from spherical cavities when the parameter

\[
\alpha = \frac{dK}{Dd\theta}
\]

is assumed constant, where \( K \) is the conductivity, \( D \) the diffusivity, and \( \theta \) the water content. Indeed the case \( \alpha \) constant is of interest as it represents soil behavior in approximate fashion. However the case of \( \alpha \) constant is only an approximation and in general the case of \( \alpha \) varying with \( \theta \) must be considered. Thus if an “average” \( \alpha \) is taken to describe the soil a recipe must be given to calculate its value when \( \alpha \) is known. This problem is totally ignored by Philip (1984).

On the other hand Parlange (1972) gave a theoretical argument to show that the value of \( \alpha \) far from the source was critical in controlling the value of the flux. It is somewhat surprising that this early paper of Parlange (1972) is ignored by Philip (1984) even though it discusses steady infiltration for more general and far more realistic soil properties.

Even when \( \alpha \) is constant the result for flux given by Philip (1984) in his Eq. [32] is too complicated. Elementary properties of Bessel functions show at once that we can describe it by the simpler expression,

\[
\phi(R) = \pi R^2 \theta, \quad \phi(R) = (n + 1) I_{n+1/2}/K_{n+1/2}.
\]

where \( Q \) is the dimensionless flux, \( R_0 \) is the argument of the Bessel functions and

\[
Q = \frac{\pi}{2R_0} \sum \left( -1 \right)^n (2n + 1) I_{n+1/2}/K_{n+1/2}.
\]

Table 1 shows the remarkable precision of Eq. [4] which is very easy to apply. Note also that Eq. [4] predicts values of \( Q \) which are too large, replacing the last term, \( 2R_0 \), by \( R_0 \), would yield values of \( Q \) which are too small. Finally when \( R_0 \) is sufficiently small Eq. [4] reduces to

\[ Q = 1 + R_0. \]

Philip (1984) obtains the above Eq. [5], his equation [33], but omits to point out that Parlange (1972) first derived that equation for exactly the same conditions as his. In addition Philip claims that the next order term is \( 0(R^2) \) while it is obvious from Eq. [4] that the next order is \( 0(R) \). The fact that he has not obtained the higher order terms correctly is probably a result of starting from an expression for \( Q \) far more complex than Eq. [2].

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References


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I am grateful to Parlange and Hogarth for comments which indicate the need for a postscript to Philip (1984c). Their discussion of solutions for variable \( \alpha \) leads to issues about which I must differ fundamentally from them, for reasons which I hope are clear in what follows. The symbolism is generally that of Philip (1984c and d).

The development of the quasilinear analysis (e.g. Philip, 1984a,b,c,d) stems from recognition of the need for a simple method of analysis of steady multidimensional unsaturated flows accurate enough for engineering purposes and those of soil physics in the field. Such a method should, if possible, employ a single parameter to characterize the capillary properties of the unsaturated soil. The concept \( \alpha \) of the quasilinear analysis fulfills that requirement.

Contrary to the view of Parlange and Hogarth that I have “totally ignored” the question of how to calculate \( \alpha \), I dealt with this in Philip (1984a,b). When the moisture potential \( \Psi \) is 0 at the water supply surface and \( \infty \) at infinity, exponential representation of \( K(\Psi) \) yields

\[
\alpha^{-1} = K(0)^{-1} \int_{0}^{\psi} d\Psi .
\]

In that case, I proposed use of Eq. [1] to evaluate \( \alpha \) for \( K(\Psi) \) arbitrary, since it “ensures a \( K(\Psi) \) representation that is at least correct in an integral sense. At the very worst (it) should ensure that quasilinear solutions yield good estimates of the integral properties of flows”. In the general case with \( \Psi = \Psi_0(0 < \Psi_0 > \Psi_1) \) at the supply surface and \( \Psi \leq \Psi_1 \leq \infty \) at infinity, Eq. [1] is replaced by

\[
\alpha^{-1} = [K(\Psi_0) - K(\Psi_1)]^{-1} \int_{\Psi_1}^{\Psi_0} d\Psi.
\]

When \( K(\Psi) \neq 0 \), we recognize the background downward flow at infinity, \( K(\Psi_1) \), and, where appropriate, subtract it out (cf. Philip, 1957, 1984c).

Equation [2] (with Eq. [1] a special case) gives optimal evaluation of \( \alpha \) for steady infiltration from the spherical cavity, in the sense that it, and no other prescription, gives the leading term of the cavity discharge, \( q \), correctly both in the limit as \( R_0 \to 0 \) and as \( R_0 \to \infty \).

The limit \( R_0 \to 0 \) corresponds to absorption (gravity negligible) from the sphere. The well-known solution (Philip, 1969) gives

\[
Q = 1 + R_0.
\]