Comments on “Laboratory Tests of a Theory of Fingering During Infiltration into Layered Soils”

Baker and Hillel (1990) obtained very interesting data on movement of water across an interface of fine over coarser soil. The following presents further interpretation of these data with regard to the physics of fingering and the condition under which wetting front instability occurs.

The linear regression of the water entry suction of the sublayer, $\psi_e$ (cm) with the median grain particle diameter, $d$ (cm) in Fig. 7 of Baker and Hillel (1990) shows that they are related as

$$\psi_e = 0.437 \, d^{-1}$$

where the constant of 0.074 in the original equation of Baker and Hillel (1990) is so small that it may be neglected. This scaling where $\psi_e$ is inversely proportional to $d$ in Eq. [1] is in agreement with the scaling theory of similar soils originally proposed by Miller and Miller (1956).

Also, based on Miller and Miller scaling, we expect that the finger width, $D$, is inversely related to medium grain diameter and obeys the relation (Parlange et al., 1990)

$$D = \frac{a \, G}{d \left(1 - \frac{Q_t}{K_a}\right)}$$

where

- $a$ = constant if the soils are similar
- $G$ = geometric factor that is equal to 1 for two-dimensional fingers and $\pi/4.8$ for three-dimensional fingers (Glass et al., 1990)
- $Q_t$ = flux entering the sublayer
- $K_a$ = the sublayer conductivity at the water entry pressure

In order to check Eq. [2], the cases presented in Fig. 5d,e, and f (Baker and Hillel, 1990) are considered first. The data presented in their Table 2 suggest that the fingers in Fig. 5e and f are approximately three-dimensional (wetted fraction of 0.13 and 0.09, respectively), while for Fig. 5d the fingers span the width of the chamber and, therefore, are two-dimensional (wetted fraction of 0.33). Applying Eq. [2] to the situations in Fig. 5d,e, and f we find that the ratio of finger width should be 2:2:1, respectively, which is approximately what is observed.

Equation [2] can be further simplified assuming again similar soils, and using data presented by Selker et al. (1989) to give the minimum size finger as (Steenhuis et al., 1990)

$$D_{\text{min}} = \frac{1}{3 \sqrt{K_a}}$$

where $D_{\text{min}}$ is in m and $K_a$ in m/d. Using Eq. [3] we find that the minimum finger width for Fig. 5d, e, and f are approximately 2.2, 2, and 1 cm, respectively. The ratio of finger width is approximately 2:2:1, which is again very close to what is observed.

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We find the comments by Selker, Parlange, and Miller to be very interesting and helpful, and offer only the following.

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In conclusion, the experiments of Baker and Hillel confirm Miller and Miller (1956) scaling, the concepts of finger formation and with the stability criterion of Raats (1973).

References


