Comments on “Scaling Water Characteristic and Hydraulic Conductivity Based on Gregson–Hector–McGowan Approach”

Ahuja and Williams (1991) discussed the application of the Gregson–Hector–McGowan (GHM) model (Gregson et al., 1987), to a range of soils in the USA. They pointed out that the GHM model represents a simple relationship between volumetric water content; \( \theta_i \) and the matric potential, \( \psi_i \), for a spatial location \( i \) within a soil type or within a group of soils as:

\[
\ln(-\psi_i(\theta)) = a_i + b_i \ln(\theta) \tag{1}
\]

and states that a linear relationship exists between \( a_i \) and \( b_i \) across locations, expressed as:

\[
a_i = p + q b_i \tag{2}
\]

and converges within a narrow region around the point \((-q,p). \) Ahuja and Williams showed that \( b_i \) is a scale factor for each soil by substituting Eq. [2] in Eq. [1] and rearranging. They demonstrated that the GHM approach was applicable to the range of soils they examined, though the values of \( p \) and \( q \) derived were different from those of Gregson et al. (1987). Classifying the soils into groups improved the assumption of a unique relationship between \( a_i \) and \( b_i \). The authors hypothesized that in “real soils” a high correlation between \( a_i \) and \( b_i \) could be due to the terms \( \ln(-\psi_i) \) and \( \ln(\theta_i) \), the logarithms of the air-entry potential and saturated volumetric soil water content, respectively, being relatively constant across a group of soils. They interpreted \(-q \) and \( p \) as effective values of \( \ln(\theta_i) \) and \( \ln(-\psi_i) \) respectively, around which all \( \psi(\theta) \) functions tend to converge.

The following analysis sheds some light on the authors’ hypothesis, with some interesting implications. If we assume the Campbell expression (Campbell, 1974) of the water retention characteristic:

\[
\frac{\psi}{\psi_\theta} = \left(\frac{\theta}{\theta_\theta}\right)^b \tag{3}
\]

and expressing Eq. [3] in the logarithmic form, we have:

\[
\ln(-\psi) - \ln(-\psi_\theta) = b[\ln(\theta) - \ln(\theta_\theta)] \tag{3a}
\]

Rearranging Eq. [3], we get:

\[
\ln(-\psi) = [\ln(-\psi_\theta) - b\ln(\theta_\theta)] + b\ln(\theta) \tag{3b}
\]

Now, let

\[
a = \ln(-\psi_\theta) - b\ln(\theta_\theta)\]

then we can rewrite Eq. [3b] as

\[
\ln(-\psi) = a + b\ln(\theta) \tag{4}
\]

which is the Brooks–Corey relationship (Brooks and Corey, 1964). Further, let us assume that there is a perfect linear relationship between \( a \) and \( b \) and generalize it for any spatial location \( i \) within a soil type or soil group. We have then:

\[
\ln(-\psi) = a_i + b_i\ln(\theta) \tag{5b}
\]

Now, rearranging the terms from the GHM model Eq. [2], we have:

\[
a_i = p + b_i q \tag{6}
\]

Generalizing Eq. [4] for any spatial location \( i \) within a group of soils, we have:

\[
a_i = \ln(-\psi_\theta) + b_i[\ln(-\theta_\theta)] \tag{6a}
\]

then by identification between terms of Eq. [6] and [6a], it can be seen that \( p = \ln(-\psi_\theta) \) and \( q = -\ln(\theta_\theta) \). Since, theoretically, \( p \) and \( q \) are constant and thus by implication the values of \( \psi_\theta \) and \( \theta_\theta \) are also constant. Hence, from this analysis of the Campbell model, there is one value of \( \psi_\theta \) and \( \theta_\theta \) for all soils within a soil type or group. Hence, in this perfect relationship, the line passes through the point \((-q,p)\).

However, in the field or in “real soils”, the values of \( \psi_\theta \) and \( \theta_\theta \) are measured with error. Hence, the work of Gregson et al. (1987) and Ahuja and Williams (1991) provide estimates of \( p \) and \( q \), and so of the true population values of \( \psi_\theta \) and \( \theta_\theta \).

Table 1 shows the average values of \( \psi_\theta \) and \( \theta_\theta \) derived from the values of \( p \) and \( q \) from Table 3 of Ahuja and Williams (1991).

A ramification of the analysis is that \( b_i \) is only a scaling factor if you do not take the Campbell model into account. As it can be seen, if you substitute the values of \( \ln(-\psi_\theta) \) and \( -\ln(\theta_\theta) \) for \( p \) and \( q \) in Eq. [4] of Ahuja and Williams, you arrive at the equality of \( \ln(\theta) = \ln(\theta) \). However, these comments do not detract from the utility of Eq. [2] in describing the soil water characteristic for use in soil water movement modeling studies.

Table 1. Average values of the air-entry potential \( \psi \) and saturated volumetric water content \( \theta \) for the soils used in

<table>
<thead>
<tr>
<th>Soil type</th>
<th>( \psi )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kPa</td>
<td>m³ m⁻³</td>
</tr>
<tr>
<td>Ozisols</td>
<td>5.45</td>
<td>0.455</td>
</tr>
<tr>
<td>Kirkland</td>
<td>3.99</td>
<td>0.430</td>
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<tr>
<td>Renfrew</td>
<td>6.25</td>
<td>0.398</td>
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<tr>
<td>Teller</td>
<td>3.35</td>
<td>0.312</td>
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<tr>
<td>Coop.</td>
<td>0.90</td>
<td>0.427</td>
</tr>
<tr>
<td>Lakeland</td>
<td>0.86</td>
<td>0.256</td>
</tr>
<tr>
<td>Norfolk</td>
<td>0.59</td>
<td>0.354</td>
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<tr>
<td>Pinnax</td>
<td>2.11</td>
<td>0.447</td>
</tr>
<tr>
<td>Pooled Australian and British soils</td>
<td>0.37</td>
<td>0.557</td>
</tr>
<tr>
<td>Bernow, 0-45-cm depth</td>
<td>3.42</td>
<td>0.207</td>
</tr>
<tr>
<td>Bernow, 45-90-cm depth</td>
<td>1.42</td>
<td>0.364</td>
</tr>
</tbody>
</table>

Received 17 Jan. 1992.

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References