Comments on “Boundary Conditions for Displacement Experiments through Short Laboratory Soil Columns”

The study of van Genuchten and Parker (1984) has already been commented on by Parlainge et al. (1985) in connection with Brenner’s solution (Brenner, 1962). In this letter I would like to point out that the material balance concept, introduced by van Genuchten and Parker (1984) to test the correctness of the solution of Lapidus and Amundson (1952), referred to as the LA solution, is faulty. The LA solution is perfect and its correctness is described in the following paragraphs using the same symbols as in van Genuchten and Parker (1984).

The LA solution can be obtained by the method of Laplace transformation without any approximation, as given by Carslaw and Jaeger (159, p. 388). This solution, however, which is obtained without approximation, should not be examined mathematically for its mass balance error. Integration of Eq. [1], from \( t = 0 \) to \( t = t \) yields the cumulative solute mass that has entered the column from the reservoir during time \( t \). The LA solution should have been checked for its performance by deriving the flux at \( x = 0 \), \( J_s(0,t) \), and evaluating \( J_s(0,t) \) under various limiting conditions as explained below. The \( J_s(0,t) \) expression of the advection–dispersion equation (ADE) involving radioactive decay has been adequately described by Shukla (1993, p. 33 and 209). The expression \( J_s(0,t) \) for a nonradioactive solute, as applicable to the LA solution, can be written as follows:

\[
J_s(0,t) = \theta C_0 \left[ \frac{D}{\pi t} \exp\left(-\frac{v^2 t}{4Dt}\right) + v \text{erf}\left(\frac{vt}{2\sqrt{Dt}}\right) \right] \tag{1}
\]

When \( v = 0 \), Eq. [1] assumes the following form:

\[
J_s(0,t) = \theta C_0 \left( \frac{D}{\pi t} \right) \tag{2}
\]

and when \( t \to \infty \)

\[
J_s(0, \infty) = \theta C_0 v \tag{3}
\]

When \( t \to 0 \), it follows from Eq. [1] that \( J_s(0,t) \to \infty \) due to the diffusional component of the flux. This implies that for small times the solute transport from the reservoir to the column, at \( x = 0 \), is mainly due to diffusion. Equation [2] shows that the diffusional flux decreases with time, which has already been discussed by Shukla and Dignam (1987). Equation [2] also shows that the diffusional flux vanishes when \( t \to \infty \). Equation [3] shows that after long times the flux at \( x = 0 \) is solely due to advection. The above arguments clearly point out that Fig. 1 of van Genuchten and Parker (1984) shows the ratio: \([\text{integrated diffusional flux}]/(\theta C_0 v t)\) at \( x = 0 \), to evaluate the correctness of \( J_s(0,t) \) for a nonradioactive solute. The LA solution should be used without any reservation. The underlying issue of Shukla’s and our analysis is that the LA solution should be used without any reservation.

It is important to note that the solution of an ADE can also be checked for its correctness by evaluating \( C(x,t) \) when \( v = 0 \) and \( t = \infty \). Equation [4] is a well-known equation as given by Crank (1975, p. 21), where the initial and boundary conditions are the same as assumed in the derivation of the LA solution.

\[
C(x,t) = C_0 \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \tag{4}
\]

Received 29 Sept. 1993.

Environmental Research and Publications
P.O. Box 79023
Garth Postal Outlet
Hamilton, ON, Canada L9C 7N6

References


