COMMENTS & LETTERS TO THE EDITOR

Comment on Number-Size Distributions, Soil Structure, and Fractals

In recent studies the relationship between aggregate/particle number-size distributions and fractal dimensions has been examined (Logsdon et al., 1996; Kozak et al., 1996). The general principle, repeated in Eq. [1], is that the number, $N$, of aggregates/particles is related to some power-law function of diameter, $r$.

$$N \propto r^{-D_f} \quad [1]$$

The power law exponent, $D_f$, is called the fragmentation fractal dimension (Turcotte, 1986).

The attraction of Eq. [1] stems from the fact that from simple, number-size data a causal connection may be made to physical processes that occur in soil. The fractal exponent can therefore offer a unique functional quantification of soil structure. In practice, however, there are a number of pitfalls of equating power-law exponents from fragmentation data with fractal dimensions. The purpose of this comment is to reemphasize these problems in the context of two recent studies (Kozak et al., 1996; Logsdon et al., 1996).

Taking our starting point as a collection of particles formed from a set of sieved aggregate sizes that have gone through the usual methods of size analysis, what does that collection tell us about the original structure, or the process by which the aggregates fractured from the larger aggregate? Perhaps more importantly is what that collection doesn't tell us about processes occurring within the parent structure. Crawford et al. (1993) answered similar questions in a series of mathematical proofs. Their analysis proves that a one-to-one correspondence between number-size distributions and fractals does not exist. Several key points are pertinent in relation to the work of Kozak et al. (1996) and Logsdon et al. (1996).

First, the power-law exponent $D_f$ in Eq. [1] is the fragmentation fractal dimension, which is a measure of how the surface area of particles scales with measurement resolution. As one of the original analyses of number-size distributions and fractals makes clear, there is no strict relation between the fragmentation fractal dimension of a distribution of aggregate sizes and the mass fractal dimension of the parent aggregate (Turcotte, 1986). The confusion of relating $D_f$ to the mass fractal dimension of the parent aggregate partly has arisen because most researchers who quote Turcotte's original analysis fail to note that a Euclidean, nonfractal aggregate was used in the model to form a fragmentation fractal dimension from a distribution of smaller fragmented aggregates.

Secondly, although a diagnostic of fractal behavior is power-law scaling, such behavior is not necessarily indicative of a structure exhibiting fractal properties. Logsdon et al. (1996) examined scale invariance of certain aggregate parameters. Kozak et al. (1996) attempted a more rigorous analysis of the size distribution data. While the basic matrix is complete, our own work and that of others (Kozak et al., 1996; Logsdon et al., 1996) revealed that the collection of data had inherent pitfalls. These pitfalls result in the inappropriate use of aggregate definitions, reporting, and presentation of results. In particular, we question the criteria for accepting or rejecting power-law exponents with fractal dimensions.

There are a number of criticisms to the approach of Kozak et al. (1996). First, their rejection criteria are too harsh. Assuming the observed power-law to be an estimate of the boundary dimension of the original aggregate, the fractal dimension of the parent aggregate which is not explicitly commented on by Kozak et al. (1996) is necessary to reject power-law exponents with values below 2. Values of the fractal dimension of the boundary of dimensional objects are limited to values between 2 and 3 (Young and Crawford, 1991; Crawford et al., 1993). Kozak et al. (1996) gave no rationale for either their acceptance of such low values or the physical meaning that these values have. Cases exist, however, where a power-law exponent derived from a number-size distribution, derived from an aggregate, can produce values <2 and >3 during the complete fragmentation of a nonfractal aggregate structure (Crawford et al., 1993). Values >3 arise from incomplete fragmentation of a nonfractal aggregate with a fractal boundary.

Third, a remaining criticism of both the Kozak et al. and Logsdon et al. (1996) studies is the acceptance that values between 2 and 3 indicate fractal structure. Assuming the observation of fractal, or nonfractal, aggregate fragmentation is an over- or underestimate of the power-law exponent to complete fragmentation (Crawford et al., 1993). That values between 2 and 3 can arise from completion of nonfractal aggregates or fractal aggregates in both cases, the observed exponent is not a fractal dimension (Crawford et al., 1993). In the case of aggregations, the case for incomplete fragmentation.

Additionally, both Logsdon et al. (1996) and Kozak et al. (1996) noted that single values of a fractal dimension vary across size aggregate classes or particle numbers. This behavior as multifractal, quoting Perfect et al. (1993) evidence. However, Perfect et al. (1993) were correct in attributing such changes in values of a fractal dimension to scales to multifractal behavior. Rather, the multifractal description of phenomena (e.g., soil structure), number of fractal exponents (Stanley, 1991).

Intuitively, the harsh nature of the physical treatments to soil to which soil is subjected in order to obtain size distributions must exclude strong causal relationships between size distributions and physical processes. Individual distributions, complex connections between aggregation, and transport through structure are destroyed by the destructive processes used. The destruction of organic matter during the particle fractionation process and the probability of a functional connection between size distributions and soil structure are orders of magnitude that which occurs in a soil profile. Add to this the probability of a functional connection between size distributions and soil structure is unknown.