Comments on “On the Physical Properties of Apparent Two-Phase Fractal Porous Media”

Hu Zhou, Edmund Perfect, Baoguo Li, and Yizhong Lu*

Yu et al. (2009) box-counted the solid and pore phases of a two-dimensional diffusion-limited aggregation (DLA) structure and observed linear relationships in the resulting log-log plots of the number of boxes ($N_\delta$) versus box size ($\delta$) over the same scale ranges for both phases. Based on this observation, Yu et al. (2009) proposed that “both the solid phase and pore phase could be fractal to form an apparent two-phase fractal porous medium in some range of statistically self-similarity.” However, we contend that the concept of a two-phase fractal porous medium is erroneous.

The notion of dimension is central to fractal geometry. Mathematically, the Hausdorff dimension ($\dim H$) should satisfy the stability property (Falconer, 2003, p. 32):

$$\dim H(E \cup F) = \max(\dim H E, \dim H F) \quad [1]$$

where $E$ and $F$ are countable sets. In practice, the box-counting dimension ($D$) is widely used as an estimate of $\dim H$ due to its relative ease of calculation. The box-counting dimension also satisfies the stability property in Eq. [1] (Falconer, 2003, p. 48). Now suppose $F$ is a two-dimensional fractal set, then $0 < D \sim \dim H F < 2$. According to Eq. [1], we can immediately see that the dimension of $F^c$ (the complementary set of $F$) must be 2, which means that $F^c$ is a nonfractal. The “dimensions” of the solid ($D_s$) and pore phases ($D_p$) estimated by Yu et al. (2009) were 1.69 and 1.89, respectively. Inserting these two values into Eq. [1] yields $1.89$ as the dimension for the union of the solid and pore phases. Since the union of these two phases is the two-dimensional plane, i.e., $\dim H (E \cup F) = 2$, the above result contradicts physical reality. This is because a two-phase fractal porous medium is not compatible with fractal theory.

Yu et al. (2009) claimed that “apparent two-phase fractals” can exist over a finite range of scales. Here we will prove that “apparent two-phase fractals” do not exist, even over a finite range of scales. Following Yu et al. (2009) (Eq. [A9a and A9b]), the scaling of the area fractions in “some range of self-similarity” is given by:

$$\varphi_s (r) \sim r^{D_s - 2} \quad [2]$$

and

$$\varphi_p (r) \sim r^{D_p - 2} \quad [3]$$

where $\varphi_s (r) = A_s (r)/A(r)$ and $\varphi_p (r) = A_p (r)/A(r)$, with $A_s (r)$, $A_p (r)$, and $A(r)$ the solid, pore, and total areas within a radius $r$, respectively. Also from Yu et al. (2009) (Eq. [A8]), the following relationship holds between the two phases for any radius $r$:

$$1 = \varphi_s (r) + \varphi_p (r) \quad [4]$$

We now set the radius equal to $\alpha r$, with $\alpha$ arbitrarily chosen so that $\alpha r$ lies within some “range of self-similarity” as proposed by Yu et al. (2009). Contrary to statements made elsewhere in Yu et al. (2009), their Fig. 2 clearly indicates that this range can be the same for both phases. Therefore, substituting $\alpha r$ into Eq. [2], [3], and [4], we obtain the following relations (Stanley, 1996):

$$\varphi_s (\alpha r) = \alpha^{D_s - 2} \varphi_s (r) \quad [5]$$

$$\varphi_p (\alpha r) = \alpha^{D_p - 2} \varphi_p (r) \quad [6]$$

and

$$\alpha^{D_s - 2} \varphi_s (r) + \alpha^{D_p - 2} \varphi_p (r) = 1 \quad [7]$$

Substituting Eq. [4] into Eq. [7] and solving for $\varphi_s (r)$ gives:

$$\varphi_s (r) = \frac{\alpha^2 - \alpha^{D_s}}{\alpha^{D_s} - \alpha^{D_p}} \quad [8]$$

Alternatively, solving for $\varphi_p (r)$ yields:

$$\varphi_p (r) = \frac{\alpha^2 - \alpha^{D_p}}{\alpha^{D_p} - \alpha^{D_s}} \quad [9]$$

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H. Zhou, B.G. Li, and Y.Z. Lu*, Dep. of Soil and Water Sciences, China Agricultural Univ., No. 2 Yuanmingyuan West Rd., Beijing, China 100193; E. Perfect, Dep. of Earth and Planetary Sciences, Univ. of Tennessee, Knoxville, TN 37996-1410. *Corresponding author (lyz@cau.edu.cn).

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