Determining the Effective Hydraulic Properties of a Highly Heterogeneous Soil Horizon

In this study, we have attempted to determine the effective hydraulic properties of a highly heterogeneous soil horizon composed of two elementary pedological volumes (EPVs). Our upscaling approach was guided by the “scaleway” approach, in which the properties of a complex system can be estimated by multiple discrete upscaling steps. This approach was tested on a data set from laboratory measurements of hydraulic conductivity at the EPV scale, while an explicit three-dimensional soil structure was considered at the horizon scale. We then formulated a decision tree to guide the choice of the appropriate upscaling method to determine the effective hydraulic conductivity at the horizon scale. In the case of low contrast between hydraulic conductivities at the EPV scale, the effective hydraulic conductivity at the horizon scale can be achieved by calculating the Wiener bounds, which requires only the proportion of the different EPVs. In the case of high contrast between hydraulic conductivities at the EPV scale, we recommend either calculating the Cardwell and Parson bounds, or performing a direct three-dimensional numerical simulation to solve Richards’ equation, which requires an explicit representation of the three-dimensional structure of the soil horizon. The Cardwell and Parsons bounds remains a good and easily available approximation. Otherwise, more accurate estimation can be obtained by numerical simulation, although this is time consuming. A decision map is proposed to help choose the best method to estimate the effective hydraulic conductivity.

Hydraulic properties are often key parameters in environmental simulations and it is usually necessary to obtain them at the horizon scale. In this context, soil horizons represent the reference soil volume in terms of soil functioning. Nevertheless, in many cases, soil horizons are heterogeneous, for example, in stony horizons (Cousin et al., 2003), cultivated horizons (Richard et al., 2001), and also specific weathering horizons like those in Albeluvisols (Diab et al., 1988; Frison et al., 2009). In these cases, the determination of hydraulic properties remains difficult. Consequently, there are two possibilities. The soil horizon can be described either by an explicit structure with distinctive hydraulic properties or by effective soil hydraulic parameters. The first possibility requires two- or, even better, three-dimensional modeling, although the latter is not always practical to carry out. The second possibility is based on the assumption that the soil horizon can be represented by a homogeneous structure if it is possible to take into account the hydraulic properties at a smaller scale. Nevertheless, the determination of effective hydraulic properties in heterogeneous horizons cannot be done by classical laboratory experiments on decimetric samples, such as the multistep outflow method (van Dam et al., 1994) or the Wind evaporation experiment (Wind, 1968). According to the IUSS Working Group WRB (2007), these heterogeneous horizons can be described as a combination of different elementary soil pedological volumes that have different chemical and mineralogical compositions and physical properties. For example, in the case of an Albeluvisol, we can distinguish ochre and pale volumes resulting from soil evolution; in the case of a cultivated horizon, compacted and uncompact ed soil clods result from mechanical stress. In this study, we attempted to determine the effective properties at the horizon scale based on the “scaleway” upscaling approach introduced by Vogel and Roth (2003). In this upscaling approach, spatial variability is considered to exist at multiple scales, and the system can be divided into multiple discrete upscaling steps. Indeed, this approach permits dealing with multiscale heterogeneities without making assumptions about the heterogeneities of the underlying structure because the latter is taken into account explicitly.
The aims of this study were threefold: (i) to determine the hydraulic properties at the scale of the soil’s elementary volume, (ii) to determine the benefits and disadvantages of the different analytical methods for upscaling, and (iii) to compare these methods with the estimation of the effective hydraulic conductivity by using a direct three-dimensional numerical simulation.

First, the hydraulic properties were determined at the elementary scale of soil pedological volumes by adapting the method proposed by Meadows et al. (2005). The second step consisted in developing different strategies to determine the effective hydraulic properties at the horizon scale. Renard and de Marsily (1997) discussed different analytical methods based on the simple calculation of bounds to estimate the effective hydraulic conductivity in heterogeneous porous media. Until now, these methods have mostly been used in petroleum engineering and hydrogeology. Moreover, recent research to determine the effective hydraulic properties of soil has often neglected natural soils and opted for simulated structures (Knudby et al., 2006; Samouëlian et al., 2007; Durner et al., 2008). In this study, we applied the analytical methods put forward by Renard and de Marsily (1997) in the context of soil science to natural soil heterogeneities and real data measurements of hydraulic properties at the local scale. To achieve this, we used an explicit representation of three-dimensional soil structure measured by electrical resistivity tomography (Frison 2008). We also tested the accuracy of analytical characteristics that can be easily computed once the structure of the hydraulic properties is known.

**Materials and Methods**

**Soil Characteristics and Structure**

The soil studied was an Albeluvisol that exhibited several horizons composed of the juxtaposition of two EPVs. We have focused on the E and Bt horizon, from the 30- to 55-cm depth. The EPVs in this horizon can be visually distinguished by their colors (ochre and pale). Their chemical and mineralogical compositions (Montagne et al., 2008) and their different modes of hydraulic functioning (Frison et al., 2009) were analyzed on clods of the two EPVs, each clod being large enough to be a representative elementary volume of the EPV. This is consistent with previous studies on E and Bt horizons (Diab et al., 1988; Wopereis et al., 1993). The pale EPVs contained more silt and the ochre EPVs contained more clay (Table 1), but the proportion of clay increased with depth inside the whole sample volume regardless of the EPV. The bulk density of the EPVs was around 1.5 g cm$^{-3}$ and was not significantly different between the two types of EPV (Table 2). Further works conducted by Frison (2008) provided the three-dimensional structure of the soil horizon and the proportion of each EPV. The characterization of this E/Bt horizon was done during autumn 2006, when no macropores were observed in the field. As a consequence, only two types of EPV, ochre EPV and pale EPV, represent the structure of the horizon. The three-dimensional structure of this heterogeneous horizon was obtained by electrical resistivity measurements (Fig. 1). A three-dimensional soil block (90 by 52 by 30 cm) with the explicit localization of the ochre and pale EPVs was obtained after a simple binary threshold of the electrical resistivity data. The threshold was chosen by comparison between the binary resistivity image of the top of the horizon and its picture from photography (Frison, 2008). The proportion of each EPV type was also calculated on this soil block: 57% for the ochre EPVs and 43% for the pale EPVs.

**Determining Hydraulic Properties at the Elementary Pedological Volume Scale**

To calculate the effective hydraulic properties at the horizon scale, experiments were first conducted at the EPV scale. Large undisturbed blocks of the E&B horizon—about 10,000 cm$^3$—were sampled when the soil was near field capacity during the autumn.
The effective hydraulic conductivity was determined with the modified van Genuchten parameterization to a \( k \)-modal form (Vogel et al., 2008):

\[
S_e(h) = \sum_{i=1}^{k} \omega_i \left( (1 + \alpha_i h)^{1-1/n_i} \right)^{-1+1/n_i}
\]  

where \( S_e(h) \), \( \omega_i \), and \( \alpha_i \) are the effective water retention, the relative weight of the different modes, and the related van Genuchten parameters (van Genuchten, 1980). In our study, \( \omega \), the volume proportion of the different EPVs, was 0.43 for the pale EPVs and 0.57 for the ochre EPVs.

**Effective Hydraulic Conductivity**

The effective hydraulic conductivity was determined with two different methods: numerical three-dimensional variably saturated flow modeling and an analytical method with the calculation of mathematical bounds. It should be noted that the analytical method consisted in fast and easy calculation compared with the numerical one, which required a numerical three-dimensional code to solve Richards’ equation.

**Numerical Simulation of the Effective Hydraulic Conductivity Calculation.** The effective hydraulic conductivity \( K_{\text{eff}}(h) \) of the E and Bt horizon was obtained by solving Richards’ equation using HYDRUS-3D (Šimůnek et al., 2007). The three-dimensional soil structure at the horizon scale (Fig. 1) was used to allocate each node (15,600 in total) of the finite element mesh to ochre or pale soil hydraulic properties. We used a modified method to solve Richards’ equation numerically.

The effective water retention curve was obtained from the additive properties of the water retention curves at the local scale, introduced by Durner (1994). This was achieved by an expansion of the modified van Genuchten parameterization to a \( k \)-modal form (Vogel et al., 2008):

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hexahedral mesh with 80,850 elements to spatially describe the soil horizon. The average size of each element was about 2 cm³, whereas the sizes of the pale and ochre EPVs ranged from some centimeters to decimeters.

Numerical simulations were the same as described by Samouëlian et al. (2007). A steady-state flow regime was simulated by applying a constant water potential \( h \) at the upper and lower boundaries so that gravity was the only driving force and the soil potential was approximately constant throughout the domain. The initial condition was a constant pressure head, whereas the vertical boundaries were considered to be no-flux boundaries because the flow was assumed to be mainly vertical. This calculation was done for 102 pressures starting from saturated \(( h = 0 \) hPa\) to unsaturated conditions \(( h = -10,000 \) hPa\). For each water potential value, the simulation time was chosen so that the steady-state flow condition was reached. The highest mass balance error that was accepted was 0.003%. Finally, we obtained \( K_{\text{eff}}(h) \), which is equal to the simulated flux.

Calculation of the Analytical Bounds. Three types of analytical bounds, namely those of Wiener (1912), Matheron (1967), and Cardwell and Parsons (1945), were calculated to estimate the hydraulic conductivity of the heterogeneous E and Bt soil horizon.

The calculation of the Wiener and Matheron bounds is based on an assumption of the spatial arrangement of the different EPVs constituting the soil horizon and takes into account their proportion. The Wiener bounds assumed a layered model structure. When the flux is parallel to the main direction of organization of the two types of EPVs, the effective conductivity at each water potential, \( m_i(h) \), is given by the arithmetic mean of the hydraulic conductivity of each EPV:

\[
m_i(h) = \frac{2}{\sum_{i=1}^{2} \omega_i K_i(h)}
\]  

where \( \omega_i \) is the volume proportion of each EPV and \( K_i(h) \) is the hydraulic conductivity of the \( i \)-th EPV at water potential \( h \).

When the flux is perpendicular to the main direction of organization of the two types of EPVs, the effective conductivity at each water potential, \( K_{\text{eff}}(h) \), is given by the harmonic mean of the hydraulic conductivity of each EPV, \( m_h(h) \):

\[
\frac{1}{m_h(h)} = \frac{2}{\sum_{i=1}^{2} \omega_i \frac{1}{K_i(h)}}
\]

In a more complex arrangement of the different types of EPV, the \( K_{\text{eff}}(h) \) of the E&B horizon is located between these two theoretical bounds:

\[
m_h(h) \leq K_{\text{eff}}(h) \leq m_a(h)
\]

In the calculation of the Matheron bound, the geometry of the porous medium is considered isotropic. In this case, the \( K_{\text{eff}}(h) \) is determined as:

\[
K_{\text{eff}}(h) = m_a(h)^{\alpha} m_h(h)^{1-\alpha} \quad \text{with} \quad \alpha = \frac{D-1}{D}
\]

where \( D \) is the spatial dimension.

Cardwell and Parsons (1945) proposed to take into account the spatial three-dimensional arrangement of the soil horizon to define the upper and lower bounds. The effective conductivity in a given direction is bounded by: (i) the arithmetic mean of the harmonic means calculated on each cell line parallel to the main flow direction (lower bound); and (ii) the harmonic mean of the arithmetic means on each slice of a cell perpendicular to the main flow direction (upper bound). If the main flow is orientated along the vertical \( z \) axis, the \( K_{\text{eff}}(h) \) is then bounded by:

\[
m_a^*(h) \left\{ m_a^*(h) \left[ m_a^*(h) \right] \right\} \leq K_{\text{eff}}(h) \leq m_h^*(h) \left\{ m_h^*(h) \left[ m_h^*(h) \right] \right\}
\]

Results

Water Retention Curves and Effective Water Retention Curve at the Horizon Scale

Water Retention Curves for the Pale and Ochre Elementary Pedological Volumes

Figure 2a presents the water retention curve estimated from evaporation experiments for 17 pale and ochre EPVs. For potentials higher than about \(-1000\) hPa, the volumetric water content was generally higher in the pale EPVs than in the ochre ones, which was in agreement with higher porosity due to biological structures (earthworm and plant roots) observed in the field in the pale EPVs. On the contrary, for water potentials lower than \(-1000\) hPa, the water content was higher in the ochre EPVs due to their higher clay content (Table 1) (Montagne et al., 2008; Frison et al., 2009).

Nevertheless, the variability in the water retention curve within the different EPVs was high. Statistical tests on the water content at water potentials of \(-10, -33, -10, -330, -500, \) and \(-1000\) hPa showed that the difference in water content between the two types of EPV was significant for water potentials equal to or higher than \(-100\) hPa and nonsignificant for water potentials equal to or lower than \(-330\) hPa. The hydraulic parameters of the three cases are summarized in Table 3.

The Effective Water Retention Curve at the Horizon Scale

According to Eq. [4], the effective water retention curve at the horizon scale must be localized inside the domain defined by the water retention curves of the pale and ochre EPVs. Because the
proportion of ochre EPVs was slightly higher (57%) than the pale EPVs (43%), the resulting effective water retention curve was closer to the ochre EPV water retention curve. Figure 2b presents the results for Case 1; the tendency was the same for the other cases but the amplitude of the contrast between the water retention curves at the EPV scale became decreasingly significant (results not shown here).

Fig. 2. Water retention curve of the horizon studied: (a) water retention curve determined by the evaporation method on eight pale elementary pedological volumes (EPVs) and nine ochre EPVs (the bold lines represent the mean curve for each type of EPV; water contents followed by the same lowercase letter in parentheses are not significantly different according to Student’s t-test); and (b) the effective water retention curve at the horizon scale for Case 1 (black line; the orange and gray lines represent the highest contrast in water retention curves at the EPV scale).

Table 3. Hydraulic conductivity parameters for the pale and ochre elementary pedological volumes (EPVs) and for the three cases studies.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated water content, cm³ cm⁻³</td>
<td>0.46</td>
<td>0.42</td>
<td>0.43</td>
</tr>
<tr>
<td>Residual water content, cm³ cm⁻³</td>
<td>0.001</td>
<td>0.016</td>
<td>0.033</td>
</tr>
<tr>
<td>Fitting parameter α, m⁻¹</td>
<td>0.23</td>
<td>1.64</td>
<td>0.55</td>
</tr>
<tr>
<td>Fitting parameter n</td>
<td>1.33</td>
<td>1.15</td>
<td>1.23</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity, m s⁻¹</td>
<td>$1.40 \times 10^{-5}$</td>
<td>$1.83 \times 10^{-6}$</td>
<td>$1.98 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Hydraulic Conductivity of Elementary Pedological Volume and Effective Hydraulic Conductivity

Hydraulic Conductivity Curves for the Pale and Ochre Elementary Pedological Volumes

Figure 3a presents the hydraulic conductivity curve for eight pale and nine ochre EPVs. Statistical tests performed on the
logarithmic value of hydraulic conductivity for water potentials equal to those already studied for the water retention curve, i.e., $-10$, $-33$, $-100$, $-330$, $-500$, and $-1000$ hPa, showed that hydraulic conductivity was always significantly different: the hydraulic conductivity was always higher in the pale EPVs whatever the water potential.

As shown in Fig. 3b, the contrast in hydraulic conductivity between the ochre and pale EPVs was different for the three cases. The difference in hydraulic conductivity between the pale and ochre EPVs was maximal for a water potential around $-1000$ hPa for Case 1 and around $-400$ hPa for Case 2. For more negative water potentials, the difference decreased slightly. For Case 3, the difference in hydraulic conductivity between the ochre and pale EPVs was negligible. To check this difference, we also calculated the surface area, defined by integral differences, between the two hydraulic conductivity curves for each case (Fig. 4). As seen in Table 4, this surface area varied by an order of magnitude between Cases 1 and 3.

The Effective Hydraulic Conductivity Curve at the Horizon Scale

The estimation of the effective hydraulic conductivity curve by the numerical simulation was assumed to be the closest to the real hydraulic conductivity and was thus considered as the reference hydraulic conductivity curve. As expected, whatever the case, the effective hydraulic conductivity curve was between the hydraulic conductivity curves of each EPV and was closer to the hydraulic conductivity of the pale EPVs (Fig. 4), although the proportion of ochre EPVs was higher. This example backs up the argument that hydraulic conductivity is, above all, correlated to the soil structure.

For Case 3 (Fig. 4c), the effective hydraulic conductivity curves estimated by the numerical simulation and calculated by the analytical bounds merged because the contrast in hydraulic conductivity was low. Contrary to Case 3, Cases 1 and 2 presented distinct effective hydraulic conductivity curves. As seen in Fig. 4a and 4b, the Wiener bounds and the Cardwell–Parsons bounds delineated a surface area inside the domain of the hydraulic conductivity curves of the ochre and pale EPVs. These domains included the effective hydraulic conductivity curve estimated by the numerical simulation. By definition, the Cardwell–Parsons domain is included inside the Wiener domain. Indeed, the heterogeneous structure is taken into account by the Cardwell–Parsons bounds. The Wiener bounds assume an extreme geometric structure of soil with a layered structure. This is in agreement with calculation of the surface areas $S$ of the two domains: $S_{\text{Wiener}} \geq S_{\text{Cardwell–Parsons}}$, regardless of the case (Table 4).

The results of our study showed that the high Wiener bound (calculation of the arithmetic mean) was closer to the numerical simulation than the low Wiener bound (calculation of the harmonic mean). This means that the general structure of the E&B horizon was more or less parallel to the water flow. This was consistent with field observations of vertical tongues of pale EPVs and image analysis observations (Cornu et al., 2007).
**Discussion**

As shown above, for the estimation of the effective hydraulic conductivity at the horizon scale, different situations could occur depending on the contrast of the hydraulic conductivity of each elementary EPV. We propose a decision tree to guide the choice of the method best adapted to estimate the effective hydraulic conductivity of a heterogeneous soil horizon (Fig. 5). First of all, for any anisotropic medium like soil, the structure must be studied roughly, for example, by qualitative soil profile observation. The two extreme cases consist in a layered porous medium, where the EPVs would be either parallel or perpendicular to the water flow. Between these two extreme structures, various possibilities of structure topology and connectivity can be considered, as is often the case for the natural soil horizon. Indeed, recent research has pointed out that the topology of the subscale structure may be of crucial importance for upscaling hydraulic conductivity (Western et al., 2001; Zinn and Harvey, 2003; Knudby et al., 2006; Samouelian et al., 2007). Our method takes into account not only the topology but also the contrast between the hydraulic conductivity curves at the EPV scale to choose the appropriate upscaling method: either estimation by numerical simulation or calculation using the analytical bounds.

1. When the contrast between the hydraulic conductivity curves of the two types of EPV is low, the effective hydraulic conductivity can be rapidly and easily estimated by the calculation of the domain defined by the Wiener bounds. This method requires only the volume proportion of the different EPVs.

2. When the contrast between the hydraulic conductivity curves of the two types of EPV is high, we recommend either estimating the effective hydraulic conductivity by numerical simulation or calculating it with Cardwell–Parsons bounds. Both methods require the three-dimensional structure of the soil horizon. An initial estimation could be given quickly by the calculation of the Cardwell–Parsons bounds. Depending on the required accuracy, this first estimation might be sufficient. Otherwise, a more accurate estimation can be provided by numerical simulation. Nevertheless, it should be noted that this numerical simulation is much more time consuming than the calculation of analytical bounds.

Nevertheless, determining an absolute value for a “high” or a “low” contrast of hydraulic conductivity at the EPV scale remains difficult. One way of deciding whether Wiener bounds can be used consists in calculating their ratio, which itself depends on the ratio between the hydraulic conductivity curves of the two types of EPV and the volume percentage of each EPV. This ratio, $R_W$, is calculated as:

$$ R_W = \frac{\frac{K_p(h)}{K_o(h)} + (1 - \omega_p) \frac{K_p(h)}{K_o(h)}}{\frac{\omega_p + (1 - \omega_p) K_p(h)}{K_o(h)}} $$

where $K_p(h)$ and $K_o(h)$ represent the hydraulic conductivities of the pale EPVs and ochre EPVs, respectively, while $\omega_p$ is the volume fraction of the pale EPV. When the contrast between the hydraulic conductivity curves is low, $R_W$ is close to one. This means that the Wiener bounds enclose a rather narrow region in which the actual effective hydraulic conductivity is located. For more complex cases, we propose a decision map based on the $R_W$ ratio to simultaneously track the effect due to the contrast between hydraulic conductivities at the EPV scale and that for each possible proportion between the two EPVs. The hydraulic conductivity contrast was extended up to 4.5 on a logarithmic scale, covering in this way the range of hydraulic properties proposed by Vogel et al. (2006) between macropores and a soil horizon. In our study, we considered that the Wiener bounds could be correctly applied when the $R_W$ value was < 3 (Fig. 6). Nevertheless, the $R_W$ threshold value has to be considered case by case, depending on (i) the accuracy of the measurements themselves at the lower scale, and (ii) the expected accuracy required for the simulation. In our survey, the Cases 2 and 3 had $R_W$ values < 3 and the use of Wiener bounds remained then acceptable. Consequently, by calculating the effective hydraulic conductivity, it is possible to avoid the difficulties related to numerical simulation. For Case 1, the $R_W$ values were from around 3 to 100; the contrast between the hydraulic conductivity curves of the pale and ochre EPVs therefore remained too
high (around 2.5 on a logarithmic scale) to estimate the effective hydraulic conductivity by using the Wiener domain. In this case, the calculation of the effective hydraulic conductivity curve by the Cardwell–Parsons bounds remained the best and most easily available approximation.

This decision tree was built with the assumption that the structure is bimodal at the horizon scale. Nevertheless, according to the “scaleway” upscaling approach introduced by Vogel and Roth (2003), the applied concept could be generalized to estimate the effective hydraulic conductivity at scale \( n \) from knowledge of scale \( n - 1 \). With respect to Wiener bounds assuming a layered structure, this suggests a simple way for upscaling to the scale of the soil profile or even the watershed. Nevertheless, this approach only allows the calculation of the vertical flux component so that this concept would be valid only when lateral flows are negligible or else can be neglected.

At the profile scale, the general structure of the soil is layered, with horizons parallel to the soil surface and generally perpendicular to the main water flow. Consequently, an easy way to estimate the effective hydraulic properties at the profile scale would be to calculate the low Wiener bound, that is to say, the harmonic mean of the hydraulic conductivity of the different superimposed soil horizons weighted by their thickness (Fig. 5).

At the small watershed scale, it can be assumed that the general organization of the pedological mantle consists of a juxtaposition of soil units. If we hypothesize that the general hydrodynamic functioning of this watershed is vertical and that the hydraulic conductivity curve of each soil unit is known, we can calculate the effective hydraulic conductivity curve of the watershed from the high Wiener bound, that is to say, the arithmetic mean weighted by the surface area of each soil unit.

**Conclusions**

In this study, we investigated the impact of using different analytical bounds to upscale the effective hydraulic properties, especially the hydraulic conductivity, of a complex horizon. The calculations of the analytical bounds were based on either the volume proportion of the different EPVs (Wiener and Matheron) or the three-dimensional structure (Cardwell–Parson), which included additional topological and connectivity information about soil structure. As already acknowledged in the literature, prior knowledge of topology and connectivity leads to more precise determination of the effective hydraulic conductivity. Because calculating analytical bounds is much easier than performing a numerical simulation based on a three-dimensional structure, however, we defined the case in which the first method would lead to satisfactory results. We demonstrated that the contrast of hydraulic conductivity between the two EPVs was crucially important for choosing the most appropriate method to estimate the effective hydraulic conductivity. Indeed, for a low contrast between these two EPVs (Case 3), it was shown that the Wiener method, which requires only the volume proportions of each EPV, provided satisfactory results. For high contrast between the two EPVs, an adequate upscaling method required the three-dimensional soil structure, i.e., topology and connectivity. For a hydraulic contrast equal to or higher than Case 3, the use of Wiener bounds is then inadvisable. The calculation using Cardwell–Parson bounds is recommended at first because it is simpler to compute. If the accuracy of the calculated effective hydraulic conductivity is not sufficient, the numerical simulation is then the most relevant method.

We then used our results to develop a decision map that can be used for other studies to help choose the appropriate analytical bounds as a function of the accuracy expected up to a conductivity contrast of 4.5 on a logarithmic scale.

Our results are based on a natural soil horizon defined by only two EPVs, but extrapolation to more than two EPVs is easy. The sole restriction is the need to define the hydraulic properties at the EPV scale. Moreover, this approach was tested on real measurements at the EPV scale combined with an explicit three-dimensional structure at the horizon scale, but it can be generalized for estimating effective hydraulic conductivity at other scales. For example, this approach could be applied to define the effective soil hydraulic...
properties for each soil unit at the watershed scale, leading to better account being taken of heterogeneous soil horizons in simulations of environmental functioning.

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