Soil Hydraulic Functions Determined from Measurements of Air Permeability, Capillary Modeling, and High-Dimensional Parameter Estimation

Prediction of flow and transport through unsaturated porous media requires knowledge of the water retention and unsaturated hydraulic conductivity functions. In the past few decades many different laboratory procedures have been developed to estimate these hydraulic properties. Most of these procedures are time consuming and require significant human commitment. Furthermore, multiple measurement techniques are typically required to yield an accurate characterization of the retention and hydraulic conductivity function between full and residual saturation. We present a more efficient and robust approach to estimating the hydraulic properties of porous media. Our method derives an optimized pore-size distribution from measurements of air permeability and using recent advances in capillary modeling and high-dimensional parameter estimation. The section diameters of different parallel capillaries representing the pore structure of a porous medium are optimized with a multi-algorithm optimization method by comparing measured and modeled air permeability values. The optimized pore model is subsequently used to predict the water retention and water permeability functions. The predicted soil hydraulic properties are shown to be in excellent agreement with experimentally determined data from an unconsolidated porous medium column. Our approach does not impose any functional form of the hydraulic properties and requires only measurements of air permeability.

Knowledge of the water retention and hydraulic conductivity functions (hereafter referred to as hydraulic properties) is essential for prediction of flow and transport through unsaturated porous media such as soils, porous rocks, and filtering systems. Many different procedures have been developed in the past few decades to determine the hydraulic properties of unsaturated soils, most of which are discussed in Chapter 3 of Dane and Topp (2002).

Yet, existing laboratory methods require considerable human commitment and resources. Moreover, multiple different experiments are typically required to provide soil hydraulic properties over the entire range of water content values. Arguably, there still is a need to develop a single procedure for the determination of soil hydraulic properties that is fast and reliable and works well for a range of soils. Such a procedure should not only be cost-efficient, but also provide important new ways to rapidly characterize the soil hydraulic properties at larger spatial scales.

We present a more efficient and robust procedure to estimate the hydraulic functions of porous media. Our method combines the strengths of capillary modeling and high-dimensional parameter estimation to derive an equivalent pore-size distribution from measurements of air permeability. Specifically, we optimize the section diameters of different parallel capillaries by minimizing the difference between measured and capillary model–predicted air permeability values. We employ recent advances in global optimization using AMALGAM evolutionary search (Vrugt and Robinson, 2007; Vrugt et al., 2009) to solve for the many hundreds of pore radii values that define the pore network. This method employs a diverse set of optimization algorithms simultaneously for population evolution and adaptively favors constituent algorithms that exhibit the highest reproductive success during the search. The AMALGAM-optimized pore-size distribution

Abbreviations: BC, Brooks–Corey–Burdine [model]; MVG, van Genuchten–Mualem [model]; TDR, time domain reflectometry.
is then used to predict the hydraulic properties of the soil sample under consideration.

**Soil Hydraulic Functions**

Practical experience suggests that determination of the hydraulic conductivity or water permeability function is far more cumbersome than estimation of the water retention curve. It has therefore become common practice to indirectly estimate the unsaturated hydraulic conductivity function from knowledge of the water retention function. Examples of this approach include the Brooks–Corey–Burdine (BC) (Brooks and Corey, 1964) and van Genuchten–Mualem (MVG) (van Genuchten, 1980; Mualem, 1976) soil hydraulic models. Because of their conceptual simplicity these models have become the standard for describing soil hydraulic functions.

Notwithstanding the progress made, recent contributions to the vadose zone hydrologic literature have questioned the continued usefulness of the classic models fitting paradigm to determine the hydraulic properties of unsaturated soils. The main problem with the BC and MVG models is that they impose strong constraints on the shape of the hydraulic functions and therefore do not always fully extract the information contained in the experimental data. A one-size-fits-all approach misses the opportunity to study specific peculiarities of individual soil samples. For instance, data points that deviate somewhat from other measurements and underlying theory are quickly dismissed and treated as outliers, but these measurements could reveal important diagnostic information about the soil under consideration.

Free-form soil hydraulic functions are designed to address some of the problems experienced with fixed functional forms, and therefore they have desirable advantages. First, the functional shape of the hydraulic functions is derived entirely from the experimental data. This minimizes conceptual uncertainty. Second, the structure and heterogeneity of the pore system is inferred from the data and does not require any prior knowledge of the pore-size distribution. Finally, an improved fit to the retention and hydraulic conductivity observations is generally observed. This improvement is likely to cause changes in the predictions of flow and transport simulators. Examples of free-form approaches to estimating the soil hydraulic functions include the work by Bitterlich et al. (2003) on piecewise polynomial functions using data from multistep outflow experiments, which was later improved by Iden and Durner (2007a); Unsal et al. (2005) and Unsal and Dane (2006), who used equivalent soil pore geometry modeling in combination with measurements of air permeability during soil drying; and Iden and Durner (2007b), using an evaporation method. For the porous medium under consideration (Berea sandstone), Unsal et al. (2005) showed substantial differences in the measured water retention curve and the one based on BC parameter estimates derived from the air permeability relationship.

Our work builds on the ideas presented in Unsal et al. (2005) and Unsal and Dane (2006), but demonstrates a much closer agreement between measured and modeled soil hydraulic functions by exploiting recent advances in global optimization. Our procedure requires only measurements of air permeability to predict the complete water retention and water permeability functions of porous media. It should be emphasized that in this study we determined not only the air permeability and water retention curves, the latter for model evaluation as in Unsal et al. (2005), but also the water permeability curves with the goal of improving model evaluation. In other words, the water retention, water permeability, and air permeability functions were determined simultaneously on the same porous medium.

**Capillary Model and Inverse Approach**

The Equivalent Pore-Size Distribution Model

A complete description of the equivalent pore-size distribution model can be found in Unsal et al. (2005) and Unsal and Dane (2006). Their pore model provides a hypothetical description of a porous medium consisting of parallel capillaries containing sections with different diameters. This makes the model more realistic than having a bundle of straight capillaries of different diameters. It also allows the model to produce wetting curves and thus describe hysteresis. The sections in the capillaries can be assigned the same height or different heights. For simplicity, here we used equal heights. The porosity of the equivalent model, $\text{por}_{\text{mod}}$, should be the same as that of the porous medium used in the experiment, $\text{por}_{\text{pm}}$. Upon calculating the total pore volume of the hypothetical model, $V_{\text{pores,mod}}$ (m$^3$), its total cross-sectional area (solids plus pore space) $A_{\text{mod}}$ (m$^2$) can be calculated from:

$$\text{por}_{\text{mod}} = \frac{V_{\text{pores,mod}}}{A_{\text{mod}} l_{\text{mod}}} \quad [1]$$

where $l_{\text{mod}}$ (m) is the height of the equivalent model sample; that is, it is equal to the summation of all section heights within a capillary. In all calculations reported herein, we used $l_{\text{mod}} = 0.1$ m, which corresponds to the height of the various soil samples under consideration. During optimization (discussed below), new values are assigned to the section diameters, which changes $V_{\text{pores,mod}}$. This requires $A_{\text{mod}}$ to be adjusted (to maintain the same porosity) as needed for volume flux density calculations to be discussed hereafter. Thus, with each update of the pore radii using AMALGAM, the value of $A_{\text{mod}}$ is calculated using Eq. [1] so that the porosity remains the same.

The model sample is assumed to be initially saturated with water. Ignoring the gravitational potential and stepwise increasing the capillary pressure head at the bottom of the model sample, we can apply the Young–Laplace equation to calculate at what capillary pressure head value the top section of a given capillary will drain.
Depending on the diameter of the immediate underlying section, it will also drain if its diameter is greater than that of the overlying section, or it will remain filled if its diameter is smaller than that of the overlying section. Additional increases in capillary pressure head will continue to drain sections within a capillary. These calculations are made for all capillaries making up the model. The volume of sections drained at each capillary pressure head value determines the water retention relation. Once a given capillary has completely drained (i.e., all sections within this capillary are void of water), it becomes available for air flow as established by an air pressure gradient across the sample (we will assume it to be unity). The volume flow rate \( (Q_j, \text{m}^3 \text{s}^{-1}) \) will be the same in all sections of a given capillary \( j (j = 1, \ldots, M, \text{where } M \text{ denotes the number of air conducting capillaries}) \) and can be described by Poiseuille’s law:

\[
Q_j = \frac{\pi r_j^4}{8\eta} \left( P_{i-1} - P_i \right) \frac{L}{ \tau}
\]  

[2]

where \( r_j \) is the radius of section \( j (j = 1, \ldots, N, \text{where } N \text{ signifies the number of sections per capillary}), P_{i-1} \) (Pa) is the pressure at the section entrance, \( P_i \) (Pa) is the pressure at the section exit, \( \eta \) denotes the air viscosity (Pa s), and \( L \) represents the pore section height (m). In Eq. [2], \( P_0 \) is the pressure at the bottom of a capillary, while \( P_N \) is the pressure at the top of a capillary. Hence for \( N \) sections per capillary we obtain \( N \) equations with \( N \) unknowns, \( Q_1 \) and \( P_1 \) through \( P_{N-1} \) being the unknowns, which allows us to solve for \( Q_j \). The total air flow rate through the model sample can then be obtained from

\[
Q_{\text{total}} = \frac{1}{\tau} \sum_{j=1}^{M} Q_j
\]

[3]

where the tortuosity factor \( \tau \) is introduced to make the flow more realistic. For a discussion of \( \tau \) and its dependency on the volumetric water content, the reader is referred to discussions of the Kozeny–Carman equation (Burdine, 1953; Wylie and Gardner, 1958a,b; Brooks and Corey, 1964; Corey, 1994). Dividing \( Q_{\text{total}} \) by \( A_{\text{mod}} \) yields a Darcy-type air flux density, \( q_a \) (m s\(^{-1}\)), as expressed in Eq. [8], which now allows us to calculate the effective air permeability, \( k_a \) of the model sample at a given water content. Repeating this process for an increasing number of drained capillaries will yield effective air permeability values as a function of water content for the model sample under consideration. These predicted values of air permeability can then be compared with their respective measured values. This allows for optimization of the pore-size distribution and requires a powerful optimization algorithm that can solve for the many hundreds of unknown section diameters of the different parallel capillaries in the least squares sense.

Similarly to calculating the effective air permeability values for the model sample, we can calculate the effective water permeability values (Unsal and Dane, 2006). These values only need to be determined once, that is, for the best possible equivalent pore-size distribution. The disadvantage of our capillary model is that we only deal with parallel capillaries (containing different section diameters). Therefore, no flow can occur from one capillary to another, and the section diameters change abruptly in diameter from one section to the next. The model can, however, account for hysteresis as drainage and wetting of pore sections occur at different capillary pressure head values. Rather than implementing a drainage sequence only, one should implement a wetting sequence following drainage. It should also be mentioned that our parameter estimation algorithm, outlined in the next section, can be applied to any pore structure model, independent of its complexity. Finally, as discussed by Corey (1994, p. 92–93) we used a variable value for \( \tau \) depending on the water content. The starting value at saturation was \( \tau = 2 \).

The AMALGAM Multimethod Parameter Estimation Algorithm

Successful application of the procedure outlined relies on the availability of a powerful optimization algorithm that can handle the many hundreds of parameters that define the section diameters of the different parallel capillaries in the pore network. Here we use recent advances in self-adaptive multimethod evolutionary search using the AMALGAM evolutionary optimization algorithm (Vrugt and Robinson, 2007; Vrugt et al., 2009). This is a global search method that employs a diverse set of optimization algorithms simultaneously for parameter estimation and adaptively favors constituent algorithms that exhibit the highest reproductive success during the search. We utilize the standard single-objective implementation of AMALGAM presented in Vrugt et al. (2009). This method is called AMALGAM-SO, and simultaneously uses different search methods for population evolution to improve search efficiency and maximize chances of finding the global minimum of our least squares objective function. Because the population of points is shared by the individual algorithms, these optimization algorithms not only learn from each other, but also build a synergy that enhances search efficiency. In this paper we use the standard implementation of AMALGAM-SO that uses the covariance matrix adaption (Hansen and Kern, 2004; Hansen and Ostermeier, 2001) evolution strategy, genetic algorithm (Deb et al., 2002; Beyer and Deb, 2001), and particle swarm optimizer (Eberhart and Kennedy, 1995; Kennedy and Eberhart, 1996) for population evolution. This particular blend of optimization algorithms has been shown to work well for high-dimensional problems and provides the basis to solve for the hundreds of radii in the capillary model. A detailed description of AMALGAM-SO can be found in Vrugt et al. (2009) and is beyond the scope of the current paper.

In all calculations reported herein we used the standard algorithmic settings of AMALGAM-SO presented in Vrugt et al. (2009) and used a population \( X \) of \( N = 100 \) different individuals with a maximum total of 5000 capillary model evaluations (50 generations). Each individual of \( X \), hereafter referred to as \( x_i \),
(j = 1,..., n), defines a pore network that consists of 25 parallel capillaries with 10 different sections each. This results in a total of n = 250 different pore radii that are subject to optimization with AMALGAM-SO. The feasible ranges of the pore radii were taken to be between 15 and 200 \( \mu \)m and were used to generate an initial population of points, \( X_0 = \{x_i^j\} \) (i = 1,..., N; j = 1,...,n) with Latin Hypercube sampling. This initial sample is subsequently improved iteratively with AMALGAM-SO by minimizing the sum of squared error (SSE) between measured \( \hat{k} \) and model-predicted \( (k_a; m^2) \) air permeability:

\[
SSE(x) = \sum_{j=1}^{m} \left[ k_j(x) - \hat{k}_j \right]^2
\]  \[4\]

A total of 250 parameters is significant to fit the air permeability data collected in this particular experiment. One could argue that this is overparameterization, and that significantly fewer parameters would be appropriate. If it comes to interpretation of the individually optimized pore radii values, we agree with this assertion, but the main interest of this paper is not on the actually calibrated pore network but rather the model-predicted water retention and water permeability functions. There are many different combinations of the pore radii that will result in similar functional shapes of the soil hydraulic functions. Estimating this uncertainty is important, but beyond the scope of the current paper. This could be explored in future work.

♦ Materials and Methods

To illustrate the usefulness and applicability of our approach to estimating the hydraulic functions of porous media from air permeability values we used data from a 1-m-long glass column (75-cm i.d.) with two Plexiglas endcaps. The column was filled as uniformly as possible with sand (particle-size distribution: 0–106 \( \mu \)m, 0.5%; 106–250 \( \mu \)m, 11.2%; 250–500 \( \mu \)m, 24.5%; 500–840 \( \mu \)m, 63.6%; 840–2000 \( \mu \)m, 0.16%). The filling process took place by consecutively adding small amounts of deionized water and sand in such a way that the sand was always completely submerged in water. The latter contained a small amount of cupric sulfate to prevent algae growth. The height of the packed sand column was 970 mm. Miniature ceramic tensiometers (cup length 25 mm, cup diameter 7 mm) connected to pressure transducers were used to measure the soil water pressure head \( (h, m) \) values at nine locations spaced 100 mm apart, starting at 100 mm from the bottom. Air pressure head values \( (h_a, m) \) were determined with pressure transducers connected to perforated stainless-steel tubes (8-mm o.d.) installed through ports located at the same heights as the tensiometers, but at the opposite side of the column. The location of the top tensiometer was taken as the reference level \( (z = 0) \) for the gravitational head (70 mm below the surface of the sand).

Volumetric water content (\( \theta \)) values were determined with a \( \gamma \) radiation system (diameter of radiation beam is 6 mm; Oostrom et al., 1998, 2002) at 20-mm intervals from \( z = 0 \) to a depth of 800 mm. The \( \gamma \) radiation source \((^{241}Am)\) and detector were moved simultaneously by means of stepper motors. Movement, measurements of radiation intensity in the form of count rates, measurements of pressure values, and data collection were all under computer control (Jiang et al., 2002). All pressure transducers (air and water) and the \( \gamma \) radiation system were calibrated before the experiment. Each time before obtaining \( b \) and \( b_a \) values, the pressure transducers were connected to standard pressure sources to check and correct for drift in the signal output. Initially, steady-state, saturated downward flow was maintained with a constant head Mariotte bottle. To control infl ow rates, the Mariotte bottle was subsequently replaced by either a high \( (\)Masterflex pump model 7523-30 with head model 7016-20 from Cole Parmer Instrument Co., Vernon Hills, IL\( )\) or low flow rate peristaltic pump (Masterflex pump model C/L from Cole Parmer Instrument Co.). The initial pump flow rate was such that a layer of water was still maintained on the soil surface. Subsequently, the flow rate was reduced step-wise, and a number of steady-state, saturated, and later unsaturated flow conditions were established. Darcy flow rates \( (q_w, m \text{s}^{-1}; Eq. [8]) \) were measured and recorded as well. The experimental setup allowed for the simultaneous determination of water retention curves, effective air permeability relations, and effective water permeability relations. Please note that the water flow was downward, while the flow of air, supplied through the bottom air port, was upward. The bottom of the column remained saturated at all times.

During the simultaneous flow of water and air, it should be noted that

\[
b = h_m + h_a
\]  \[5\]

where \( h_m \) (m) is the matric head. Hence, during applied air flow, \( h_a \) needs to be accounted for to obtain \( h_m \) values. For convenience, we elected the use of capillary pressure head \( (h_c = -b_m) \) rather than matric head. The \( b_c - \theta \) relations (water retention) were determined by matching the \( b_c \) and \( \theta \) values for all tensiometer locations.

Hydraulic conductivity values were calculated from the Darcy equation:

\[
q_w = -K_w \frac{\Delta H}{\Delta z}
\]  \[6\]

where \( K_w \) (m s\(^{-1}\)) denotes the hydraulic conductivity, \( H = h + z \) (m) is the hydraulic head, and \( z \) (m, negative below reference level) represents distance. For our calculations we used the intervals between the tensiometers; that is, \( \Delta z = 100 \) mm. Once the

\[
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\]
hydraulic conductivity values were known, they were converted to effective water permeability \( (k_w; \text{m}^2) \) values using

\[
K_w = \frac{\rho_w g k_w}{\mu_w} \tag{7}
\]

where \( \rho_w \text{ (kg m}^{-3} \text{)} \) is the water density, \( g \text{ (N kg}^{-1} \text{)} \) is the gravitational force per unit mass, and \( \mu_w \text{ (Pa s)} \) is the water viscosity. To obtain the \( K_w \) and \( k_w \) values as a function of \( \theta \), the \( \theta \) values between each set of tensiometers were averaged.

Effective air permeability \( (k_a; \text{m}^2) \) values were calculated from (Ball and Schjønning, 2002)

\[
q_a = -\frac{k_a \Delta P_a}{\mu_a \Delta z} \tag{8}
\]

where \( \mu_a \text{ (Pa s)} \) signifies the air viscosity and \( P_a \text{ (Pa)} \) is the air pressure. As for the \( k_w-\theta \) relations, the \( \theta \) values were averaged for each interval to obtain \( k_w-\theta \) relations.

**Results and Discussion**

The bottom air port (800-mm depth) was used to supply air to the column to establish air flow. Data analysis was therefore performed on the top seven intervals. Because the results for all seven intervals illustrate similar findings, we only graphically show the results for the 300- to 400-mm interval (Fig. 1). Results of the other depths are summarized in Table 1. Figure 1A shows the measured effective air permeability values (plus signs) and the results based on the capillary model combined with AMALGAM-SO (solid line). The model predictions provide an excellent fit to the observed data with a RMSE of about 1.32 \( \mu \text{m}^2 \). This positive result was to be expected, however, because 250 parameters were allowed to vary in the capillary model to match the predicted air permeability values as closely and consistently as possible with the measured values. It is worthwhile mentioning the relative efficiency of AMALGAM-SO, which required only 5000 model evaluations to result in a good calibration. Additional function evaluations did show a further decline of the objective function, but such improvements were rather marginal compared to the reduction in SSE in the first 50 generations with AMALGAM-SO. These improvements also mainly considered the smoothness of the function (discussed in more depth below).

To check whether the optimization results are strongly dependent on the flexibility of the pore structure, we performed separate optimizations with a different number of capillaries and cross sections. When increasing the number of pore radii, quite similar results were obtained. The current selection of 25 capillaries with 10 sections each was a pragmatic choice to minimize as much as possible the complexity of the calibration problem. Also, our pore network

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Table 1. Root mean squared error of mismatch between measured and predicted effective air permeability, water retention, and effective water permeability functions for the seven different soil samples considered herein. Results correspond to a maximum of 5000 model evaluations with AMALGAM-SO.

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Air permeability (( \mu \text{m}^2 ))</th>
<th>Water retention (cm)</th>
<th>Water permeability (( \mu \text{m}^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–100</td>
<td>3.40</td>
<td>1.45</td>
<td>6.39</td>
</tr>
<tr>
<td>100–200</td>
<td>2.95</td>
<td>2.67</td>
<td>15.12</td>
</tr>
<tr>
<td>200–300</td>
<td>2.01</td>
<td>NA†</td>
<td>14.19</td>
</tr>
<tr>
<td>300–400</td>
<td>1.32</td>
<td>2.76</td>
<td>3.14</td>
</tr>
<tr>
<td>400–500</td>
<td>2.73</td>
<td>1.66</td>
<td>7.92</td>
</tr>
<tr>
<td>500–600</td>
<td>2.37</td>
<td>2.20</td>
<td>12.49</td>
</tr>
<tr>
<td>600–700</td>
<td>3.29</td>
<td>4.20</td>
<td>14.72</td>
</tr>
</tbody>
</table>

† NA, Tensiometer–pressure transducer–signal conditioner unit quit functioning.

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![Fig. 1. Comparison of model-predicted (A) air permeability, (B) water retention, and (C) water permeability functions (solid lines) against their respective measured values (plus signs). The pore network in the capillary model consists of 250 different parameters, and their values are determined by calibration against the measured air permeability data using a maximum of 5000 model evaluations with AMALGAM-SO. The water retention and water permeability measurements are not used for calibration and only used to demonstrate that our approach yields accurate and robust soil hydraulic functions.](image_url)
needs to mimic a representative elementary volume, and our setup with 250 parameters was deemed adequate for this.

To demonstrate that the optimized pore network is not overfitted, but closely represents the underlying properties of the soil, consider Fig. 1B and 1C, which present a comparison of measured (plus signs) and capillary model-predicted (solid lines) water retention (Fig. 1B) and effective water permeability (Fig. 1C) functions. The predictions of the capillary model closely correspond with the experimentally determined data. An overparameterized pore network is likely to exhibit poor predictive performance, particularly when evaluated against another data type to which it was not calibrated. The edginess of the predictions is attributed to the abrupt drainage of complete pore sections. The resulting nondifferentiability of the soil water retention curve not only makes it difficult to derive the corresponding (tabulated) values of the soil water capacity function, but also impairs convergence of Newton–Raphson-type numerical solution schemes in vadose zone flow and transport models.

There are various ways in which we can smooth the fit to the experimental data and ensure differentiability of the respective soil hydraulic functions. The simplest approach is to increase the number of parallel capillaries and allow for a closer match with the experimental data. A more parsimonious approach would be to add a penalty term to the objective function that promotes differentiability of the underlying function and, hence, ensures smoothness. In this case, one could use the multiple objective AMALGAM optimizer of Vrugt and Robinson (2007) and select from the nondominated set that solution that exhibits the most acceptable tradeoff in the fitting of the experimental data and minimization of the penalty term.

Although our initial pore network was not based on any preconceived distribution, the AMALGAM-SO optimized pore radii exhibit a lognormal distribution (data not shown). This is consistent with the behavior of many naturally varying parameters, as demonstrated by Brutsaert (1966), D’Hollander (1979), Tuli et al. (2001), and Kosugi (1996). To support our notion that water permeability functions using parameters based on water retention data are often inadequate, we present these parameters as estimated from both the experimentally determined water retention and water permeability relations in Table 2. Theoretically the values in Columns 1 and 2 and in Columns 3 and 4 should be similar. This is obviously not the case and strengthens our case of using free-form relations.

In this study we were only concerned with the predictive ability of the optimized pore structure that was derived by calibration against the air permeability data. It is particularly important to assess uncertainty on the final estimates of the pore radii, yet this is particularly difficult to estimate given the significant model nonlinearity, high parameter dimensionality, and considerable parameter interaction. Approximate parameter confidence intervals can be estimated using a classical first-order approximation in the vicinity of the best pore-radii value. Such local linear intervals are unfortunately often misleading in the presence of considerable model nonlinearity and parameter correlation (Vrugt and Bouten, 2002). On the contrary, Markov chain Monte Carlo simulation approaches are especially designed to derive exact confidence intervals of the parameters, but our experience with such methods on nonlinear problems of similar dimension suggests that many millions of function evaluations are required to converge to the target distribution. Whereas adaptive samplers such as DREAM are more efficient, this still requires parallel computing and remains outside the realm of the current work. Indeed, the scope of the current study is to demonstrate the ability of a simplified pore network to predict the water retention and water permeability function. This will only require access to the best values of the parameters. Confidence intervals for the pore radii and corresponding uncertainty estimates of the soil hydraulic functions will be the subject of future endeavors.

The results for the other intervals (Table 1) closely correspond to those presented in Fig. 1 for the 300- to 400-mm depth. The predicted soil hydraulic functions at each interval closely match the experimental data, with average prediction errors (RMSE) of air permeability, water retention and water permeability that are quite similar for the different depths. These findings provide strong support for the claim that our pore network optimization approach yields accurate functions of the soil hydraulic properties. Our method does not impose any functional form and requires only measurements of air permeability. Note that our selection of 25 different capillaries with 10 sections each was based on pragmatic rather than theoretical considerations. Future work will analyze the relationship between prediction error and dimensionality of the pore network.

Although in our experimental procedure we used a γ radiation system to determine the volumetric water content values, other
types of measurements, like time domain reflectometry (TDR), are entirely acceptable. Application of TDR, in addition to measuring air permeability in the field (Jalbert and Dane, 2003) opens new avenues of determining field hydraulic properties that should be much faster than existing field methods.

Summary and Conclusions

We presented a procedure to predict the hydraulic properties of porous media. The procedure combines knowledge of the experimentally determined effective air permeability function, an assumed equivalent pore-size distribution model, and the optimization scheme AMALGAM-SO. The latter is used to optimize the equivalent pore-size distribution so that calculated effective air permeability values match the measured values as closely as possible. The ensuing pore-size distribution is then used to predict the water retention and effective water permeability functions. The newly developed procedure was applied to seven intervals (samples) of a long, unconsolidated, porous medium column. The predicted soil hydraulic functions showed excellent agreement with experimentally determined data. A strong aspect of our procedure is that it does not require any preconceived shapes of the hydraulic conductivity functions. If applied to field situations, it could speed up the determination of field soil hydraulic properties considerably.

Of course, the air permeability method would be subject to similar limitations as water permeability methods, such as the heterogeneous nature of field soils. The advantage for air permeability is that steady-state flow conditions will be obtained in the order of seconds rather than hours or days as for water permeability methods. Although a measurement of air permeability will be quick, it may take considerable time, depending on soil type, to allow another measurement at a different, uniform, water content. It should be mentioned that the field air permeameter as developed by Jalbert and Dane (2003) operates over rather shallow depths.

Acknowledgments

We gratefully acknowledge the useful comments of the two anonymous reviewers and Associate Editor, Insa Neuwinger, who greatly improved the current version of our manuscript. The source code of AMALGAM-SO was written in MATLAB. Sequential and parallel implementations of this software can be obtained from Dr. Vrugt (jasper@uci.edu) on request. Dr. Vrugt was located at the Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA, where he was supported by a J. Robert Oppenheimer Fellowship from the LANL postdoctoral program, while Dr. Unsal operated the equivalent pore-size distribution model, and the optimization scheme AMALGAM-SO. The latter is used to optimize the equivalent pore-size distribution so that calculated effective air permeability values match the measured values as closely as possible. The ensuing pore-size distribution is then used to predict the water retention and effective water permeability functions. The newly developed procedure was applied to seven intervals (samples) of a long, unconsolidated, porous medium column. The predicted soil hydraulic functions showed excellent agreement with experimentally determined data. A strong aspect of our procedure is that it does not require any preconceived shapes of the hydraulic conductivity functions. If applied to field situations, it could speed up the determination of field soil hydraulic properties considerably. Of course, the air permeability method would be subject to similar limitations as water permeability methods, such as the heterogeneous nature of field soils. The advantage for air permeability is that steady-state flow conditions will be obtained in the order of seconds rather than hours or days as for water permeability methods. Although a measurement of air permeability will be quick, it may take considerable time, depending on soil type, to allow another measurement at a different, uniform, water content. It should be mentioned that the field air permeameter as developed by Jalbert and Dane (2003) operates over rather shallow depths.

References


