The Effect of Contact Angle on Saturation Overshoot

For certain porous media and initial conditions, constant flux infiltrations show a saturation profile that exhibits overshoot. This overshoot, which is the cause of gravity-driven fingering, cannot be described by standard models of unsaturated flow, and it is likely controlled by the exact nature of the pore filling at the initial front. These frontal dynamics have recently been studied as a function of infiltrating flux and infiltrating fluid and were present in our earlier paper. The overshoot behavior is found to disappear below a certain flux which has been named the overshoot flux. We found that, except for water, the overshoot flux $q_{over}$ is a linear function of surface tension $\sigma$ divided by the viscosity $\mu$, $q_{over} \sim \sigma/\mu$. This is expected if the physics at the front that controls overshoot are the same physics that controls the capillary desaturation curve, but the overshoot flux for water did not fall on this curve. To resolve this discrepancy, we conducted additional experiments to test how wettability affects the saturation overshoot.

We have conducted additional experiments using a water–ethanol mixture (Table 1) in a further attempt to resolve the overshoot flux discrepancy for water. Just as with all of the fluids besides water, the overshoot flux of the water–ethanol mixture scales with the surface tension divided by the viscosity (Fig. 1).

The effect of viscosity, surface tension, miscibility, vapor pressure, and density on the overshoot flux was tested in the previous work (Aminzadeh and DiCarlo, 2010). In this note, we test how wettability affects the overshoot flux. The wettability of the sand to each of the imbibition fluids was explored using a method proposed by Bachmann et al. (2000). This method involves producing a monolayer of the 30/40 sand (the working sand in the infiltration experiments with $d_{50} = 0.5$ mm) using a large piece of double-sided adhesive transparent tape, and placing a drop of the working fluid on the sand, and observing the behavior of the drop. When any of the alkanes or alcohols were placed on the sand, these fluids completely spread on the rough surface of the sand. However, when pure water was tested an apparent macroscopic contact angle was observed.

To measure the actual value of the microscopic contact angle, we subsequently performed capillary rise experiments using both water and a water–alcohol mixture that completely spread on the sand. We filled two identical columns (50 cm long with diameter of 0.95 cm) with 30/40 sand and connected the bottom of the columns to a constant head tank of water and water–ethanol mixture. In the first experiment we measured the liquid rise in each column using light transmission and in the second experiment by weighing the imbibed fluid and using mass balance. After capillary/gravity equilibrium (over 3 days), we measured a capillary rise of $h_w = 3.4 \pm 0.1$ cm for water and $h_m = 2.5 \pm 0.05$ cm for the water–ethanol mixture.

From these measurements, we calculated the contact angle of water by assuming that the contact angle of the mixture is zero $\theta_m = 0^\circ$ (Bachmann et al., 2006), and that the capillary rise is given by

$$h_w = \frac{2\sigma_w \cos \theta_w}{\rho_w g r}$$  \[1\]

where $\theta_w$ is the contact angle for water, and $r$ is a characteristic pore size. Since the same sand was used for both fluids, we obtain for the water

$$\cos \theta_w = \frac{\sigma_m}{\sigma_w} \frac{\rho_w h_w}{\rho_m h_m}$$  \[2\]
from which we calculated a contact angle of water equal to \( \theta_w = 30 \pm 2^\circ \). The physical properties of water and mixture are given in Table 1.

Along with affecting the capillary rise, the contact angle will affect the size and thus the transport capacity of the wetting layers. For simplicity, we use a triangular cross-section to model each pore and to capture the effect of contact angle on the layer flow (Fig. 2).

We assume that each corner of the pore has a layer of fluid, and the area of each layer can be calculated as

\[
A = CR^2
\]  

where \( R \) is the radius of the circular arc of the gas–liquid interface, \( C \) is a constant (for a given medium and injecting fluid) given by

\[
C = \frac{\cos \left( \alpha + \theta \right) \cos \theta}{\sin \alpha} - \frac{(90 - \alpha - \theta)}{360} \frac{2\pi}{\beta}
\]

where \( \alpha \) is the half angle of the corner (Blunt and Scher, 1995). Using \( \alpha = 30^\circ \) for an equilateral triangle and \( \theta_w = 30^\circ \) for water, we found for the same radius of curvature the area available to flow for water is 0.5 times less than it is for the entirely wetting fluids.

The flux through this layer can be calculated using the results of Ransohoff and Radke (1988) model. They derive an equation for fluid movement through corners of capillary tube as a function of pore geometry and fluid properties and found that

\[
Q = \frac{R^2 A dP_c}{\mu \beta dz}
\]

where \( Q \) is the flow rate in one layer, \( P_c \) is the capillary pressure, \( A \) is the area of corner, and \( \mu \) is the viscosity of the injected fluid. \( \beta \) is a dimensionless flow resistance that they solved for using numerical solutions of Stokes flow. This flow resistance scales as \( \beta \sim C^{-1} \); they reported calculated values of \( \beta = 54.1 \) for water with contact angle of \( 30^\circ \) and \( \beta = 31.1 \) for completely wetting fluids.

For a particular porous medium, we assume that the radius of the curvature \( R \) depends only on the size \( (D) \) and roughness \( (\lambda) \) of the medium and is independent of the fluid. Thus, the flow rate in one layer in Eq. [5] can be turned into an overall flux in a bundle of layers through \( q \sim Q(D)^2 \), and using \( P_c \sim \sigma/R \), the overall layer flux should scale on the fluid properties through

\[
q \sim \frac{C \sigma}{\beta \mu}
\]

where, as before \( \sigma/\mu \) is the ratio of capillary to viscous forces, but now \( C/\beta \) is the geometrical factor that takes in the account of wettability or non-zero contact angles. We label this ratio as the “corrected surface tension viscosity ratio”.

Figure 3 shows the overshoot flux plotted versus \( C/\beta \) \( \sigma/\mu \) for all of the fluids used. We observed that the contact angle correction brings the overshoot flux \( q_{\text{over}} \) of water in line with that of the other completely wetting fluids. This is an encouraging result, and it shows that the heuristic assumptions on a competition between flow through wetting layers and flow through the bulk gives simple predictions of overshoot flux that match for many different fluids.

In summary, the results presented in this technical note suggest that the overshoot flux (and the related transition between a sharp
and diffuse wetting interface) is controlled by the flow of wetting fluid through wetting layers ahead of the main front. We find that the relevant fluid properties that control the flux through these layers are the viscosity, surface tension, and contact angle. This suggests that the flow through the layers ahead of the wetting front controls the sharpness of a wetting front, and the associated overshoot in gravity driven displacements.

References


