Original Research

Core Ideas

• The accuracy of the PI method in estimating soil $K_s$ was tested by numerical simulations.
• Estimated $K_s$ using two-ponding-depth and multiple-ponding-depth infiltration were compared.
• Transient and steady-state infiltration data for six soils were used to estimate $K_s$.
• The PI should yield more accurate $K_s$ estimates in coarse- than fine-textured soils.
• The transient method does not solve the $K_s$ inaccuracy problems in fine-textured soils.

Accuracy of Saturated Soil Hydraulic Conductivity Estimated from Numerically Simulated Single-Ring Infiltrations

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The single-ring pressure infiltrometer (PI) method is widely used to determine saturated soil hydraulic conductivity, $K_s$, directly in the field. The original and still most common way to analyze the data makes use of the steady-state model developed by the Canadian School in the 90s and two (two-ponding-depth, TPD, approach) or more (multiple-ponding-depth, MPD, approach) depths of ponding. The so-called Wu method based on a generalized infiltration equation allows analysis of the transient infiltration data collected by establishing a single ponding depth of water on the infiltration surface. This investigation, making use of simulated infiltration runs for initially unsaturated sand to silty clay loam soils, showed that, with a run duration of practical interest (e.g., 2 h), the PI can be expected to yield more accurate estimates of $K_s$ in coarse-textured soils than in fine-textured soils even if the transient method is used instead of the steady-state method. Performing a three-level experiment and analyzing the estimated steady-state infiltration rates with both the TPD and MPD approaches is a way to predict the reliability of the estimated $K_s$ value. The $K_s$ accuracy should be acceptable if the two approaches yield similar results. Otherwise, the MPD approach should be expected to yield more accurate $K_s$ estimates than the TPD approach. The transient method does not solve the $K_s$ inaccuracy problems in fine-textured soils because obtaining accurate $K_s$ data requires that the portion of total infiltration varying linearly with time represent a high percentage of total infiltration, but this percentage is small in fine-textured soils when the run does not exceed a few hours. This investigation opens some new perspective on the use of infiltration data to make predictions on the expected reliability of the $K_s$ calculations with reference to both steady-state and transient data analysis procedures.

Abbreviations: L, loam; LS, loamy sand; MPD, multiple-ponding-depth; PI, pressure infiltrometer; S, sand; SAL, sandy loam; SCL, silty clay loam; SL, silt loam; TPD, two-ponding-depth.

Saturated soil hydraulic conductivity, $K_s$, should be determined in situ for interpreting and simulating soil hydrologic processes since, in this case, the disturbance of the sampled soil volume is minimized and its functional connection with the surrounding soil is maintained (Bouma, 1982). Due to the high spatial variability of this soil property, a large number of individual determinations of $K_s$ should be performed to characterize an area of interest at a given time (Reynolds and Zebechuk, 1996; Mallants et al., 1997; Bagarello et al., 2013a). The duration of a single run cannot be excessively long because the required field work should be practically sustainable.

In the last 30 yr, field soil hydraulic conductivity characterization by permeameters and infiltrometers has become very common, mainly thanks to the theoretical and practical developments by the Canadian school (Reynolds and Elrick, 1985, 1987, 1990, 1991; Reynolds et al., 1985, 1992). In particular, the single-ring pressure infiltrometer (PI) (Reynolds and Elrick, 1990) has been used in many investigations (Vauclin et al., 1994; Ciullaro and Lamaddalena, 1998; Bagarello and Iovino, 1999; Angulo-Jaramillo et al., 2000; Bagarello et al., 2000, 2013b, 2014; Reynolds et al., 2000; Mertens et al., 2002; Bagarello and Sgroi, 2004; Gómez et al., 2005; Verbist et al., 2009, 2010, 2013). The PI method uses a ring with a small radius that is inserted into the soil to a short depth. A constant depth of ponding, $H_f$, is established within the infiltration ring, and the flow rate...
into the soil is monitored. Three-dimensional, steady, ponded flow out of the ring is used to determine \( K_s \) by a model that takes hydrostatic pressure, capillarity, and gravity components of flow out of the ring into account (Reynolds and Elrick, 1990). The two-<sup> </sup>(TPD) and multiple-ponding-depth (MPD) approaches can be applied to determine \( K_s \) and the so-called matric flux potential, \( \phi_m \), using exclusively steady-state infiltration rates. The two approaches differ by the number of ponded depths of water that are established in succession on the infiltration surface, i.e., two or more than two.

Flow from infiltrometers into unsaturated soil goes through an initial transient phase of decreasing rates and then approaches steady state (Elrick and Reynolds, 1992b). Therefore, applying the steady-state model of Reynolds and Elrick (1990) needs the collection of reliable steady-state infiltration rate data. The equilibration time, or the time to near-steady-state conditions, depends on many factors. In particular, it generally increases with finer soil texture, drier initial soil conditions, and increasing depth of water ponding on the infiltration surface, along with the depth of cylinder insertion and ring radius (Reynolds et al., 2002a, 2002b; Reynolds, 2008). According to Reynolds and Elrick (2002), a reasonable estimate of steady flow, or quasi-steady flow, can generally be considered acceptable in natural environments and only early-time transient flow procedures are practicable if equilibration times are particularly long, i.e., many hours or even days.

It is not easy to assess if the estimated steady-state infiltration rates are reliable during a field application of the PI method. As a matter of fact, the criteria that can be applied to detect steady-state conditions are unavoidably approximate, and different criteria could yield a contrasting conclusion on the time to steady state (Bagarello et al., 1999; Bagarello and Giordano, 1999). In other words, in practical use of the method, a run of fixed duration does not assure attainment of a steady-state flow rate under any circumstance, but examining the data could be not enough to recognize that steady state was not reached. Therefore, it is necessary to establish what happens when the steady-state model of Reynolds and Elrick (1990) is used to analyze an infiltration run for which steady-state conditions are unavoidably estimated on the basis of the collected data. Working under initially more or less dry soil conditions has a particular interest since equilibration times are expected to be particularly long in this case but the run duration cannot exceed limits that are dictated by practical and environmental constraints. Indeed, only a few investigations making use of very long field runs can be found in the literature (e.g., Papanicolaou et al., 2015; Alagna et al., 2016).

A way to overcome equilibration time issues could be applying the so-called Method 1 by Wu et al. (1999) (referred to as the Wu method in the following) that uses the whole infiltration curve without discriminating between early-time and steady-state infiltration. This method has received little testing after its development (Reynolds and Elrick, 2002) and some of the tests that were made need to be complemented by other checks. In particular, Wu et al. (1999) successfully verified their method against simulated infiltration data. However, the run duration was 1 h for a sandy soil but 1 d for a sandy clay loam soil and 10 d for a clay soil. Testing the method against shorter runs is of more practical interest. Moreover, contrasting information can be found in the few studies dealing with the performance of this method. For example, Bagarello et al. (2009) and Verbist et al. (2010) concluded that the Wu method was a valid alternative to steady-state methods of analysis, but Di Prima et al. (2018) recognized that steady-state approaches performed better than the Wu method, mainly as a consequence of the difficulty in fitting the infiltration model to the data.

Numerical simulation of an infiltration process into an initially unsaturated porous medium is a powerful tool to test hypotheses and to check factors affecting the applicability of a particular analytical procedure to estimate soil characteristics (Bagarello et al., 2013b). For example, numerically simulated data were used by Wu et al. (1993) to explain erratic \( K_s \) estimates obtained by the borehole permeameter technique in soils with macropores and abrupt layers. Lai and Ren (2007) and Lai et al. (2010) used numerical simulation to improve the determination of \( K_s \) by the double-ring infiltrometer. Dušek et al. (2009) and Dohnal et al. (2016) analyzed numerically generated single-ring data to test the dependence of the infiltration rate on several factors, such as ring diameter, ring insertion depth, and ponding depth of water on the infiltration surface. Bagarello et al. (2013b) used numerical simulation to test the performance of the TPD approach for PI data collected in heterogeneous soils. Numerically simulated data were used by Reynolds (2013) to assess different borehole infiltration analyses for determining \( K_s \) in the vadose zone. Bagarello et al. (2016) used numerical simulation of a single-ring infiltration process to test the single-level, steady-state analysis developed for the PI (Reynolds and Elrick, 1990).

In practical application of the PI method, the run duration is generally fixed in advance, and it is in most cases short or relatively short since it does not exceed a few hours at the most. The available water volume for the run is also fixed in advance, especially for applications in remote areas, and even in this case the run duration has to be expected to be rather short. Possibly, the collected data will allow an estimate of the steady-state flow rate, but they will also appear to be analyzable with a method that does not distinguish between early-time and steady-state infiltration. Establishing what is the best way to determine \( K_s \) in such situations could improve routine use of the PI method in the field.

The general objective of this investigation was to check procedures for determining \( K_s \) with numerically simulated single-ring PI runs for a variety of soils. The specific objectives were (i) to test the performances of both steady-state and transient methods of analysis, and (ii) to verify if the collected infiltration data can be used to make a prediction about the reliability of the estimated \( K_s \).

**Theory**

The approximate analytical expression for steady, ponded flow out of a ring into rigid, homogeneous, isotropic, uniformly unsaturated soil is (Reynolds and Elrick, 1990)
\[ Q_s = \frac{r}{G} (K_s H + \phi_m) + \pi r^2 K_s \]  

where \( Q_s \) [L³ T⁻¹] is the steady-state flow rate, \( r \) [L] is the ring radius, \( K_s \) [L T⁻¹] is the saturated soil hydraulic conductivity, \( H \) [L] is the steady depth of water ponding in the ring, and \( \phi_m \) [L² T⁻¹] is the matric flux potential, defined as (Gardner, 1958)

\[ \phi_m = \int_h^0 K(b)db \quad -\infty \leq b \leq 0 \]

where \( K(b) \) [L T⁻¹] is the soil hydraulic conductivity vs. pressure head, \( b \) [L], relationship and \( b_1 \) [L] is the initial pressure head. The dimensionless shape factor, \( G \), takes into account the complex interactions between ring radius, depth of ring insertion, \( d \) [L], depth of ponding in the ring, soil capillarity, and gravity. The values of the shape factor were found to be nearly independent of soil hydraulic properties and \( H \) for \( H \geq 0.05 \) m. Therefore, the following relationships was developed to estimate \( G(G_s) \):

\[ G_s = 0.316 \frac{d}{r} + 0.184 \quad [3] \]

Two different approaches can be used to simultaneously determine \( K_s \) and \( \phi_m \) of Eq. [1], depending on the number of the applied steady depths of water ponding during the experiment (Reynolds and Elrick, 1990).

The TPD approach requires the steady-state flow rates, \( Q_{s1} \) and \( Q_{s2} \), corresponding to two ponding depths of water, \( H_1 \) and \( H_2 \) (\( H_2 > H_1 \)), consecutively established on the infiltrating surface, i.e., without occurrence of a drainage phase in the passage from \( H_1 \) to \( H_2 \). If only the effect of the ring radius and the insertion depth on the shape factor is accounted for by Eq. [3], the following relationships can be applied to calculate \( K_s \), \( \phi_m \), and the so-called \( \alpha^* \) [L⁻¹] parameter:

\[ K_s = \frac{G}{r} \left( \frac{Q_{s2} - Q_{s1}}{H_2 - H_1} \right) \]

\[ \phi_m = \frac{G}{r} \left( \frac{H_2 Q_{s1} - H_1 Q_{s1} - \pi r G_c Q_{s2} - Q_{s1}}{H_2 - H_1} \right) \]

\[ \alpha^* = \frac{K_s}{\phi_m} \]

The MPD approach uses two or more \( H \) levels and Eq. [1] written in the form

\[ Q_s = \left( \pi r^2 k_s + \frac{r \phi_m}{G_c} \right) + \frac{rK_s}{G_c} H \]

Therefore, the slope, \( b_1 \), and the intercept, \( b_0 \), of the linear least-squares regression line through a plot of \( Q_s \) vs. \( H \) allow the two unknowns to be obtained:

\[ K_s = \frac{b G_c}{r} \]

\[ \phi_m = \frac{G_c}{r} \left( b_0 - \pi r^2 K_s \right) \]

Equation [6] can then be used to estimate \( \alpha^* \). The MPD experiment is expected to have an advantage over the TPD experiment. In particular, with the former approach, random variability in \( Q_s \) due to measurement error or small-scale heterogeneity tends to be filtered out by the regression line (Reynolds and Elrick, 1990). This filtering is probably not very important when the PI run is numerically simulated, but it may have interest if the data are collected under real field conditions.

The Wu method (Wu et al., 1999) is based on the assumption that the following cumulative infiltration curve can be used to describe the infiltration process:

\[ I = A_w t + B_w t^{0.5} \]

where \( I \) [L] is cumulative infiltration, \( t \) [T] is time and \( A_w \) [L T⁻¹] and \( B_w \) [L T⁻⁰⁵] are the curve parameters. Equation [10] is fitted to the \((a, I)\) data pairs measured from the beginning of the PI experiment to obtain an estimate of \( A_w \) and \( B_w \). Then, \( K_s \) is given by

\[ K_s = \frac{\Delta \theta \sqrt{(H + G^*)^2 + 4G^* C - (H + G^*)}}{2T_c} \]

where \( \Delta \theta \) [L³ L⁻³] is the difference between the saturated volumetric soil water content, \( \theta_s \) [L³ L⁻³], and the initial volumetric soil water content, \( \theta_i \) [L³ L⁻³] and the \( G^* \) [L], \( C \) [L], and \( T_c \) [T] terms have the following expressions:

\[ G^* = d + \frac{r}{2} \]

\[ C = \frac{1}{4 \Delta \theta} \left( \frac{B_w}{b} \right)^2 \frac{a}{A_w} \]

\[ T_c = \frac{1}{4} \left( \frac{B_c}{b A_w} \right)^2 \]

where \( a \) and \( b \) are dimensionless constants (\( a = 0.9084, b = 0.1682 \)). An estimate of the \( \alpha^* \) parameter may be obtained by the following relationship:

\[ \alpha^* = \frac{K_s}{\phi_m} \approx \frac{K_s}{\phi_m} \]

where \( \phi_m^* \) [L² T⁻¹] is the matric flux potential calculated with a modified van Genuchten hydraulic conductivity–pressure head function. The Gardner (1958) hydraulic conductivity function was considered by Reynolds and Elrick (1990) but, according to Eq. [13], the choice of the hydraulic conductivity function doesn’t have a significant impact on the calculation of \( \alpha^* \). The \( \phi_m^* \) term is given by

\[ \phi_m^* = K_s^2 T_c \]
Materials and Methods
Soils and Numerical Simulations

Numerical simulations were performed for the six homogeneous soils considered by Hinnell et al. (2009). Soil hydraulic properties were modeled according to the van Genuchten–Mualem model (Mualem, 1976; van Genuchten, 1980), with hydraulic parameters taken from Carsel and Parrish (1988) (Table 1):

\[
\Theta(h) = \frac{\theta_s - \theta_i}{\theta_s - \theta_i} = \left(1 + \alpha_{vG} b h^m\right)^{-1/m} \quad m = 1 - \frac{1}{n} \quad [15a]
\]

\[
K(h) = \alpha^* \left[1 - \left(1 - \theta_i/m\right)^{1/m}\right]^{2} \quad [15b]
\]

where \( \Theta \) is the effective saturation, \( h \) [L] is the soil water pressure head, \( \theta_s \) [L^3 L^3] is the volumetric soil water content, \( \theta_i \) [L^3 L^3] and \( \theta_r \) [L^3 L^3] are the saturated and residual volumetric soil water contents, respectively, \( \alpha_{vG} \) [L^3], \( m \), and \( n \) are soil-specific empirical parameters of the van Genuchten–Mualem model, \( K \) [L T^-1] is the unsaturated soil hydraulic conductivity, and \( K_s \) [L T^-1] is the saturated soil hydraulic conductivity. These soils were considered to be appropriate for studying a three-dimensional infiltration process into an initially unsaturated porous medium from a small-size source (Hinnell et al., 2009). Moreover, the selected soils differed widely by their hydraulic properties, allowing exploration of a wide range of situations, including those where equilibration times of the ponding infiltration run are expected to be rather long (Reynolds, 2008).

A two-dimensional axisymmetric vertical flow domain was adopted for all the simulations using the HYDRUS-2D/3D software package (Šimůnek et al., 2007), which is widely used for simulating water, heat, and/or solute movement in two or three dimensions (Shan and Wang, 2012; Chen et al., 2015; Bautista et al., 2016; Rezaei et al., 2016). The dimensions of the flow domain were 50 cm in the \( x \) direction and 100 cm in the \( z \) direction and 100 cm in the flow domain. A constant head of water was established in the middle of the flow domain, i.e., 30 by 50 cm, was adopted for the SIL and SCL soils to ensure the simulations could run properly with the finest possible mesh size to improve computation efficiency and reduce the mass balance errors. Moreover, a variable mesh size of finite elements was used that was generated using the MESHGEN subroutine embedded in the HYDRUS-2D/3D software package. A mesh size of 0.05 cm was adopted for the zone from 0 cm (surface) to the 3-cm depth of the flow domain. The mesh size was then gradually increased to a maximum of 2.64 cm (or 0.41 cm for the SCL soil) for the deeper part of the flow domain.

For a given soil and an established initial soil water content, different PI runs were simulated by considering a ring having a radius \( r \) = 75 mm. For this reason, a notch was generated 75 mm away from the origin in the \( x \) direction on top of the flow domain to represent the infiltrometer wall. The notch was 5 mm in width and 30 mm in depth, which is equal to the insertion depth of the ring. A ring insertion depth \( d \) = 0 mm was also considered for some simulations. Both the outer border and the bottom of the flow domain were far away from the possible wetting zone. Therefore, both the lateral and bottom boundaries had negligible effect on the infiltration process. A constant head of water was established for the upper boundary condition of the infiltration zone, whereas the bottom condition of the flow domain was free drainage. The boundary condition on the infiltrometer wall (notch) and on the unconfined soil surface was no flux. A uniform distribution of the soil water content in the form of a water head was selected for the initial conditions of all the simulations.

A variable time step was adopted to reduce the computation time while assuring simulation accuracy. The initial, minimum, and maximum time steps were 0.01, 0.01, and 60 s, respectively. The absolute water content tolerance was 0.0005 (0.05%), the absolute pressure head tolerance was 0.5 cm, and the maximum number of iterations allowed during any time step was 10. With the chosen spatiotemporal discretization, the mass balance error was <0.203% for most cases, but it did not exceed 1.08% for all simulations used in this investigation. Haverkamp et al. (1977) compared different numerical schemes and considered as “good” or “excellent” mass balance errors in the range from 0.2 to 0.4%. Reynolds (2010, 2011) obtained mass balance errors of ≤0.05% and concluded that this level of error was acceptable because it was well below the commonly accepted upper limit of 0.1 to 1% for soil and groundwater flow. Therefore, our simulations were considered acceptable according to the existing evaluation criteria (Celia et al., 1990; Rathfelder and Abriola, 1994; Jacques et al., 2006).

Three different sets of infiltration data were numerically simulated in this investigation. In particular, the 3Hd30 dataset was developed by considering a ring insertion depth \( d \) = 30 mm and three constant heads of water—\( H_1 = 50 \) mm, \( H_2 = 100 \) mm, and \( H_3 = 200 \) mm—that were established in succession, without any allowed drainage in the passage from one \( H \) value to the next. For the H100d30 dataset, \( d \) was equal to 30 mm and the single depth of ponding was \( H = 100 \) mm. For the H50d0 dataset, \( d = 0 \)

<table>
<thead>
<tr>
<th>Soil</th>
<th>( \theta_s )</th>
<th>( \theta_i )</th>
<th>( \theta_r )</th>
<th>( \alpha_{vG} )</th>
<th>( n )</th>
<th>( m )</th>
<th>( K_s )</th>
<th>( \Theta_1 )</th>
<th>( \Theta_2 )</th>
<th>( \alpha^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.045</td>
<td>0.43</td>
<td>0.145</td>
<td>2.68</td>
<td>0.627</td>
<td>297.0</td>
<td>0.0263</td>
<td>0.0265</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loamy sand</td>
<td>0.057</td>
<td>0.41</td>
<td>0.124</td>
<td>2.28</td>
<td>0.561</td>
<td>145.9</td>
<td>0.0260</td>
<td>0.0263</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.065</td>
<td>0.41</td>
<td>0.075</td>
<td>1.89</td>
<td>0.471</td>
<td>44.2</td>
<td>0.0201</td>
<td>0.0203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loam</td>
<td>0.078</td>
<td>0.43</td>
<td>0.036</td>
<td>1.56</td>
<td>0.359</td>
<td>10.4</td>
<td>0.0144</td>
<td>0.0145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silt loam</td>
<td>0.067</td>
<td>0.45</td>
<td>0.020</td>
<td>1.41</td>
<td>0.291</td>
<td>4.5</td>
<td>0.0112</td>
<td>0.0112</td>
<td></td>
<td></td>
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<tr>
<td>Silty clay loam</td>
<td>0.089</td>
<td>0.43</td>
<td>0.010</td>
<td>1.23</td>
<td>0.187</td>
<td>0.7</td>
<td>0.0117</td>
<td>0.0117</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† \( \theta_s \), residual volumetric soil water content; \( \theta_i \), saturated volumetric soil water content; \( \alpha_{vG} \), \( n \), and \( m \), soil-specific empirical parameters of the van Genuchten–Mualem model; \( K_s \), saturated soil hydraulic conductivity; \( \alpha^* \) parameter; \( \Theta_1 = (\theta_i - \theta_r)/(\theta_s - \theta_i) \), where \( \theta_i \) (m^3 m^-3) is the initial or antecedent volumetric soil water content.
and $H = 50 \text{ mm}$ were considered. A given $H$ value was maintained for 120 min because this run duration was considered to be an acceptable compromise between the need to obtain reliable estimates of the steady-state flow rate and the fact that excessively long runs could be of reduced interest, being impractical in the field (Reynolds et al., 2000; Mertens et al., 2002; Verbiest et al., 2013). Two different values of the initial effective saturation ($\Theta_1 = 0.05$ and 0.4, i.e., dry and intermediate conditions, respectively) were considered for each simulated run.

Calculations and Data Analysis

A soil was characterized by a theoretical value of the saturated hydraulic conductivity and two theoretical values (one for each $\Theta_1$ condition) of both the matric flux potential and the $\alpha^*$ parameter. For a given $\Theta_1$, the theoretical $\phi_m$ value was calculated by numerical integration of Eq. [2]. The water content range from initial water content to saturation was subdivided into $10^5$ equally spaced intervals and pressure head values, $b_{i-1}$ and $b_i$ ($i = 1, 2, ..., 10^5$), corresponding to the extremes of each interval calculated by Eq. [15a]. The trapezium rule was then used to calculate the area under the hydraulic conductivity curve by means of an automated Microsoft Excel routine.

Cumulative infiltration volumes $[L^3]$ and flow rates $[L^3 T^{-1}]$ obtained by numerical simulations were transformed into infiltrated depths of water, $I$ (mm), and infiltration rates, $i_1$ (mm h$^{-1}$), respectively, by dividing volumes by the infiltration surface, since reasoning in terms of $I$ and $i_1$ is more practical to manage infiltration data. Consequently, the steady-state flow rate obtained by Eq. [1] was also transformed into a steady-state infiltration rate by the $i_s = Q_s/(\pi r^2)$ relationship. The numerically simulated cumulative infiltration curves were used to obtain an estimate of $i_s$ by considering the last 20 min of the process with a given $H$ and determining the slope of the linear regression line fitted to the $(I, t)$ data pairs (Bagarello et al., 1999).

Simultaneous calculation of $K_s$ (mm h$^{-1}$) and $\phi_m$ (mm$^2$ h$^{-1}$) was performed using both the TPD approach with $H_1 = 50$ mm and $H_2 = 100$ mm and the MPD approach with $H_1 = 50$ mm, $H_2 = 100$ mm, and $H_3 = 200$ mm to check the performance of the two tested approaches, although they are expected to yield similar results under ideal conditions (Reynolds and Elrick, 1990). The estimated $K_s$ values were compared with the corresponding theoretical values. According to the accuracy criterion of Reynolds (2013), the estimates were deemed accurate when they fell within the range $0.75 \leq K_s / K_s^{\text{true}} \leq 1.25$ (i.e., <25% error).

The Wu method was tested by considering cumulative infiltration for $H = 50$ mm and $d = 30$ mm (the H50d30 scenario; dataset extracted from the previously developed 3Hd30 dataset), $H = 100$ mm and $d = 30$ mm (H100d30), and $H = 50$ mm and $d = 0$ (H50d0). The $r$, $d$, and $H$ values for the first two scenarios were consistent with those considered by Wu and Pan (1997) for checking their infiltration model (60 $\leq r \leq 200$ mm, 20 $\leq d \leq 100$ mm, and 0 $\leq H \leq 200$ mm). The null insertion depth of the ring considered for the third scenario was close to the lowest considered value of $d$ (Wu and Pan, 1997) and was included in this check due to the implications for the infiltration process (Dohnal et al., 2016). More clearly, infiltration is initially confined and then unconfined with $d > 0$, but it is always unconfined with $d = 0$. Estimation of $A_w$ and $B_w$ was performed by the least squares optimization technique of nonlinear fitting of Eq. [10] on the $(I, t)$ data set (Wu et al., 1999; Lassabatère et al., 2006), although this technique does not allow undoubted establishment of whether Eq. [10] fits the $I$ vs. $t$ data adequately (Vandervaere et al., 2000). Linearization of the infiltration model was proposed as a way to check the adequacy of Eq. [10] (Smiles and Knight, 1976; Vandervaere et al., 2000), and, in a field test of the Wu method on a sandy loam soil, linearization yielded similar results to those obtained by nonlinear fitting (Bagarello et al., 2009). However, linearization was checked in this investigation only for the H50d0 scenario because, using numerically simulated infiltration data, Dohnal et al. (2016) showed that, with an insertion depth $> 0$, a single straight line does not describe the entire infiltration process. The quality of the fit was evaluated by calculating the relative error, $Er$:

$$
Er = \frac{\sum_{i=1}^{k} (I_{Ec} - I_{Em})^2}{\sum_{i=1}^{k} I_{Em}^2}
$$

where $k$ is the number of considered data points, $I_{Ec}$ is the estimated cumulative infiltration, and $I_{Em}$ is the numerically simulated cumulative infiltration. According to Lassabatère et al. (2006), $Er \leq 3.5\%$ denotes a satisfactory fitting of the model to the data. Equation [10] was fitted to the $(I, t)$ data by considering different durations for each run to also see if the field test could conveniently be shortened when a transient method of analysis is applied. In particular, the considered durations were 120, 90, 60, and 30 min. Therefore a total of 144 ($A_w$, $B_w$) data pairs were obtained (3 scenarios $\times$ 6 soils $\times$ 2 initial soil water conditions $\times$ 4 run durations). Equations [11–14] and [6] were used to calculate $K_s$ and $\alpha^*$ for each considered infiltration process.

Results and Discussion

Theoretical Values of the $\alpha^*$ Parameter

The considered discretization of the $K(h)$ function allowed an accurate estimation of $\phi_m$, given that the pressure head interval was always $\leq 36.3$ mm. The theoretical $\alpha^*$ values determined by Eq. [6] varied from 0.0112 to 0.0265 mm$^{-1}$ for soils with $0.7 \leq K_s \leq 297$ mm h$^{-1}$ (Table 1). According to Reynolds and Elrick (1990) and Elrick and Reynolds (1992a), $\alpha^* = 0.012$ to 0.036 mm$^{-1}$ is expected for coarse- and medium-textured soils having $K_s$ values that range between 3.6 and 360 mm h$^{-1}$. A lower $\alpha^*$ value, i.e., 0.004 mm$^{-1}$, should be typical of fine-textured soils with $K_s = 0.036$ mm h$^{-1}$, which is approximately 20 times lower than the lowest $K_s$ value considered in this investigation. Therefore, this check of the theoretical $\alpha^*$ calculations did not reveal unexpected values.
Steady-State Method

Both the TPD and the MPD approach yielded simultaneously positive $K_s$ and $\alpha^*$ values for each soil-\(\Theta_i\) combination (Table 2). In particular, $\alpha^*$ varied from 0.00036 to 0.048 mm\(^{-1}\) with the TPD approach (<0.001 mm\(^{-1}\) only with reference to the SCL soil and $\Theta_i = 0.05$) and from 0.0036 to 0.036 mm\(^{-1}\) with the MPD approach. Simultaneously positive $K_s$ and $\alpha^*$ values denote a successful experiment (Reynolds and Elrick, 1990), but only the runs yielding both positive results for both variables and $\alpha^*$ values ranging from 0.001 to 0.1 m\(^{-1}\) should be considered reliable according to a later suggestion by Reynolds and Elrick (2002). With this more stringent criterion, the only possible uncertainty in the reliability of the $K_s$ and $\alpha^*$ calculations was detected with reference to a two-level experiment performed on the finest soil under the driest initial conditions.

Excluding the SCL soil, accurate estimates of $K_s$ (0.75 $\leq K_s \leq$ 1.25) were always obtained regardless of the considered data analysis procedure and the initial soil water content. This result was consistent with the conclusion of Reynolds and Elrick (1990) that a similar $K_s$ accuracy level has to be expected with the TPD and MPD approaches, and it was a consequence of the similarity between the $i_s$ values estimated at the end of the simulated run, $i_s\text{ES}$, and the predicted $i_s$ values by Eq. [1] and [3], $i_s\text{RE}$, i.e., according to the steady-state model of Reynolds and Elrick (1990) (i.e., differences never exceeding 5%). In other terms, each $i_s\text{ES}$ value was close to the expected steady-state infiltration rate because steady-state conditions were achieved at the end of the infiltration run. The reason why $\Delta i_s = 100(i_s\text{ES} - i_s\text{RE})/i_s\text{RE} \leq 5\%$ was considered small calls for an explanation. With reference to the porous medium–\(d–r–H\) combinations of Reynolds and Elrick (1990, their Table 2), using Eq. [1] and two alternative $G$ factors, i.e., the numerically calculated $G$ value reported in that table and the corresponding estimate by Eq. [3] yielded $i_s$ values differing by up to 17%. Therefore, $\Delta i_s \leq 5\%$ was small because this value was appreciably lower than a negligible 17% according to the analysis developed by Reynolds and Elrick (1990).

For the SCL soil, the estimates of $K_s$ were accurate with the MPD approach ($K_s = 0.80$ and 0.94 for $\Theta_i = 0.05$ and 0.4, respectively) but not with the TPD approach ($K_s = 0.12$ and 0.56). To establish the reason why the two approaches had a different performance, it was initially recognized that, for this soil, large $\Delta i_s$ values were generally obtained. In particular, $\Delta i_s$ decreased from 60.5 to 24.8% in the passage from $H_1$ to $H_3$ for $\Theta_i = 0.05$ and from 33.4 to 15.9% for $\Theta_i = 0.4$. Therefore, $i_s$ was overestimated in general, denoting that the runs were too short to allow attainment of near-steady-state conditions.

Time to steady state is known to increase with the depth of water ponding (Reynolds, 2008), which could not appear consistent with an $i_s$ overestimation decreasing with larger $H$ values. However, it should be considered that, for $H = 50$ mm, $i_s$ was estimated by a 120-min infiltration run but the $i_s$ estimate corresponding to a larger value of $H$ (e.g., $H = 200$ mm) was obtained with a longer run (120 × 3 = 360 min). In other terms, overestimation of $i_s$ decreased with $H$ because the run duration increased. To test this explanation, the $i_s$ overestimation detected for $H = 100$ mm with the three-level experiment (32% for $\Theta_i = 0.05$ and 18% for $\Theta_i = 0.4$) was compared with that associated with a 4-h steady performed with $H = 100$ mm (28 and 16% for $\Theta_i = 0.05$ and 0.4, respectively). Corresponding $\Delta i_s$ values were similar but they were slightly higher in the former case. This result was consistent with the fact that 2 of the 4 h were performed with a smaller $H$ level in the former case. Another check was made by comparing, for $H = 100$ mm, the $i_s$ overestimation associated with the three-level experiment with that obtained with an infiltration process of 2 h established on a SCL soil maintained at $\Theta_i = 0.05$ and $\Theta_i = 0.4$. An appreciably higher overestimation of $i_s$ (57% for $\Theta_i = 0.05$ and 34% for $\Theta_i = 0.4$) was detected in this last case, further confirming that the initial run with a smaller water level implied less $i_s$ overestimation for the larger $H$ level. Therefore, the suggested explanation for an overestimation of $i_s$ decreasing in the passage from the lowest to the highest $H$ values was supported by the checks that were performed.

A noticeable improvement of the calculated $K_s$ values was detected when the MPD approach was applied instead of the TPD one, especially for $\Theta_i = 0.05$, indicating that even large overestimations of $i_s$ for all established ponded depths of water did not prevent accurate calculations of $K_s$. Moreover, two overestimated $i_s$ values yielded inaccurate results (TPD approach) but exactly these two estimates plus an additional, still overestimated, $i_s$ value (MPD approach) adjusted the $K_s$ calculation.

Little faith should be placed in a good result obtained by a fortuitous cancellation of errors (Smettem et al., 1995), which means that the PI method should not be applied, regardless of the experimental and data analysis procedures (TPD or MPD), if there is a sound suspect that steady state was not reached at the end of

<table>
<thead>
<tr>
<th>Soil</th>
<th>$\Theta_i$</th>
<th>TPD approach</th>
<th>MPD approach</th>
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<tr>
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<td>$\alpha^*$</td>
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the run (Reynolds and Elrick, 2002). However, explaining why the applied approach had a noticeable impact on the reliability of the $K_s$ estimates was expected to improve our knowledge about the potential and limitations of the PI method. Equations [4] (TPD approach) and [8] (MPD approach) have exactly the same mathematical form, only difference being that the $\Delta Q/\Delta H$ gradient is calculated with reference to two successively ponded depths of water in the former case and to all established ponded depths of water (three in this investigation) in the latter case. An error-free calculation of $K_s$, i.e., $K_s = 1$, needs using a $\Delta Q/\Delta H$ gradient of 169 mm² h⁻¹ for the SCL soil. With the TPD approach, this gradient was underestimated by 88.5% for $\Theta_i = 0.05$ and 44.1% for $\Theta_i = 0.4$. With the MPD approach, underestimation decreased to 20.3 and 5.9%, respectively (Fig. 1). Therefore, the reliability of the estimates was better for the three-level experiment because the addition of a third $i$ value reduced the discrepancy between the expected and the estimated $\Delta Q/\Delta H$ gradient for both values of the antecedent soil water content.

According to this investigation, performing a three-level experiment and analyzing the estimates of $i$ with both the TPD and MPD approaches should be recommended for practical use of the PI method. If the two approaches yield similar estimates of $K_s$, we can be rather confident that near-steady-state conditions were established, and that the estimate of $K_s$ is accurate. Establishing more than two ponding depths of water was not necessary to accurately estimate $K_s$, but this information was not available before making the experiment. If an appreciably lower $K_s$ value is obtained with the TPD approach compared with the MPD one, then we should think that the latter approach yielded a more accurate estimate of $K_s$ than the former one but also that overestimation of $i$ occurred for the established $H$ levels. In other terms, the MPD approach yields the estimate of $K_s$. The comparison between this approach and the TPD one allows establishment of whether the $K_s$ calculations are based on reliable estimates of steady-state flow rates.

**Transient Method**

The infiltration model, i.e., Eq. [10], described well the data because $E_r$ did not exceed 1.2% for the H50d30 scenario ($N = 48$ runs, 6 soils × 2 initial soil water conditions × 4 run durations), 1.6% for the H100d30 scenario, and 0.6% for the H50d0 scenario. However, a good fit did not necessarily imply that the $K_s$ estimates were accurate.

In particular, the estimates of $K_s$ were poor for the three finest textured soils, i.e., the loam (L), SIL, and SCL, for the two $\Theta_i$ values when the simulated durations were 0.5 to 2 h (Table 3). In these cases, $K_s < 0.75$ was generally obtained but, in some cases (SCL soil, H50d30 and H100d30 scenarios, run durations ≤1.5 h), estimating $K_s$ was not possible because the estimated $A_w$ parameter was equal to zero, which denoted the inability of Eq. [10] to describe the data.

On the other hand, the $K_s$ estimates were generally accurate for the three coarsest textured soils, i.e., the sand (S), LS, and sandy loam (SAL), since $0.75 < K_s < 1.25$ was obtained for 56 of the 72 considered runs (3 soils × 3 scenarios × 2 $\Theta_i$ values × 4 durations), i.e., in 78% of the cases. The H100d30 scenario performed better than the other two scenarios because $K_s$ was accurately estimated in 92% of the cases (22 out of 24) with the former scenario and in 71% of the cases in the latter ones. The run duration was unimportant in terms of $K_s$ accuracy in 9 of the 18 considered cases (3 soils × 3 scenarios × 2 $\Theta_i$ values), whereas reducing the duration of the run determined a worsening (improvement) of the $K_s$ estimates in seven (two) cases. An effect of $\Theta_i$ was not detected with reference to the H100d30 scenario. For the other two scenarios, a long or relatively long run (1.5–2.0 h) yielded accurate results when the S soil was initially dry but not when it was wet. For the SAL soil, a shorter run (1.5 h) than the longest one was usable to obtain accurate data only under wet conditions.

Therefore, with a run duration not exceeding 2 h, the Wu method did not yield accurate $K_s$ values in fine-textured soils (L, SIL, SCL) regardless of the applied experimental methodology and initial wetness conditions. Estimates of $K_s$ were generally accurate in coarser textured soils (S, LS, SAL). Run success percentages did not change by varying the ring insertion depth (H50d30 and H50d0 scenarios), but, for a given ring insertion depth, more hydrostatic pressure for the run was beneficial for accurately estimating $K_s$ in these soils (H50d30 and H100d30 scenarios) because it determined a reduced sensitivity to both initial wetness and run duration compared with the other tested scenarios.

Figure 2 shows the infiltration curves for the two 2-h runs yielding the most (LS soil, $\Theta_i = 0.05$) and the least (SCL soil, $\Theta_i = 0.05$) accurate estimates of $K_s$ with the H50d0 scenario. In this case, the ring insertion depth was null and therefore an impediment to linearization of the cumulative infiltration curve (Dohnal et al., 2016) was not expected. Plotting the data on an $I$ vs. $t$ plot...
did not reveal any particular anomaly, and both runs appeared analyzable by fitting Eq. [10] to the data. However, the $I_i^{0.5} \text{ vs. } t^{0.5}$ relationship was practically linear for the LS run but not for the SCL run because the relationship was necessarily linear in this case. This last result suggested that Eq. [10] was not appropriate to describe infiltration in the fine-textured soil because soil capillarity controlled the first 2 h of infiltration (Cook and Broeren, 1994; Angulo-Jaramillo et al., 2016).

This finding suggested a test of the hypothesis that a link could be established between the reliability of the $K_s$ estimate and the relative importance of the two terms in Eq. [10], i.e., $A_w t$ and $B_w t^{0.5}$. For each scenario, Fig. 3 shows $K_s$ vs. the relative weight of the $A_w t$ term on total infiltration, i.e., $(A_w t)/(A_w t + B_w t^{0.5})$, at the end of the infiltration run. A weight of $A_w t$ that does not reach approximately the 75 to 82% of total infiltration, depending on the scenario, does not allow accurate estimates of $K_s$ to be obtained that are too low compared with the true values. A weight greater than 97 to 98% of the total infiltration implies an unacceptable overestimation of $K_s$, i.e., $K_s > 1.25$. Therefore, the estimates of $K_s$ are accurate for a weight of the $A_w t$ term, varying between 75 to 82 and 97 to 98%, and the considered scenario has a minor impact on the accuracy range because $K_s$ vs. the relative weight of the $A_w t$ term relationships obtained for the three considered scenarios showed clear similarities (Fig. 3). According to this investigation, it seems possible to make a prediction of the expected reliability of the $K_s$ estimate during a run of pre-established duration.

Estimating the $A_w$ and $B_w$ parameters of the infiltration model during the run is not a particularly complicated task, even during a field run. The weight of the two terms of the model varies with the run duration (Fig. 4), which implies that, at least to a certain degree and in some situations, it could be possible to adjust the duration of the run in an attempt to obtain a weight of the $A_w t$ term of 82 to 97%, assuring, at least in the context of this investigation, accurate estimates of $K_s$.

This investigation could explain why the estimates of $K_s$ obtained by Wu et al. (1999) were all in an acceptable range according to those researchers. Probably this conclusion was correct because their $K_s$ values were obtained from much longer simulations durations for the finer soils. In any case, an infiltration run of one or more days is of limited interest for practical application of the device, which suggests that, according to this investigation, the Wu method appears usable with good expected results in coarse to relatively coarse soils but not in fine soils.

In this investigation, more accurate estimates of $K_s$ were generally obtained with a steady-state approach than a transient approach.
Fig. 2. Representation of the infiltration data obtained with the H50d0 scenario (depth of ponding $H=50$ mm and ring insertion depth $d=0$ mm) for the initially dry (initial effective saturation $\Theta_i=0.05$) loamy sand (LS) and silty clay loam (SCL) soils on plots of cumulative infiltration, $I$ (mm), vs. time, $t$ (h), $I/t^{0.5}$ vs. $t^{0.5}$ (cumulative linearization method), and $I$ vs. $t^{0.5}$.

Fig. 3. Ratio $K'$ between the estimated and the true saturated soil hydraulic conductivity, $K_s$, against the weight of the $A_w \times t$ term on total infiltration for different scenarios and durations of the run. $H$ is the ponding depth and $d$ is the ring insertion depth.
Performing a three-level experiment and analyzing the estimated steady-state infiltration rates with both the TPD (first two levels) and MPD (all levels simultaneously) approaches is a way to predict the reliability of the estimated \( K_s \) value. Attainment of near-steady-state infiltration rates for each water level and, consequently, accurate \( K_s \) calculations are signaled by similar estimates of \( K_s \) with the two approaches. In this case, the three-level experiment is only useful to make a decision about the reliability of \( K_s \) given that the estimate of \( K_s \) was already accurate with a two-level experiment. If the estimated \( K_s \) value is substantially lower with the TPD approach than the MPD one, the expectation should be that the latter approach performed better than the former one because the gradient of steady-state flow rates with \( H \) was closer to that expected but also that steady-state conditions were not reached at the end of the infiltration runs.

The transient method does not solve the \( K_s \) inaccuracy problem in fine-textured soils because obtaining accurate \( K_s \) data requires that the portion of total infiltration varying linearly with time represents a high percentage of the total infiltration, but this percentage is small in fine-textured soils when the run does not exceed a few hours. Analyzing infiltration in real time, i.e., while performing the run, could allow establishment of the relative contribution of the two terms of the used infiltration model up to a given instant and therefore adjustment of the run duration so as to improve the accuracy of the \( K_s \) determinations, at least to a certain degree.

This investigation opens some new perspective on the use of infiltration data to make predictions about the expected reliability of \( K_s \) calculations with reference to both steady-state and transient data analysis procedures. It also suggests that it should be possible to make choices about the most appropriate run duration to obtain good quality data with the transient analysis procedure. To fully utilize the advantage of the transient method, data for longer infiltration processes should be considered for finer soils. Such measurements can conveniently be obtained by an automated system such as a pressure transducer plus a datalogger. The experimental conditions simulated in this investigation were rather wide or typical in terms of soils, initial soil water content, ring radius and insertion depth, established depths of ponding, and run duration. However, not all possible cases were considered, and working with other, experimentally plausible scenarios appears advisable in order to hopefully reinforce the conclusions of this investigation.

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References
Infiltration measurements for soil hydraulic characterization. Springer, Cham, Switzerland. doi:10.1007/978-3-319-31788-5


