Evaluating the Influence of Specimen Preparation on Saturated Hydraulic Conductivity Using Nuclear Magnetic Resonance Technology

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A series of laboratory tests were performed to investigate the influences of specimen preparation on pore size distribution of soil and saturated hydraulic conductivity ($K_s$). Nuclear magnetic resonance technology was used to measure the pore size distribution of the saturated samples of silty soil, which were prepared by three different kinds of methods: Proctor compaction, static compaction, and the consolidation method. The $K_s$ of the samples was measured by the falling head permeability test. The results show that the difference in $K_s$ caused by different specimen preparations can be large as one order of magnitude, as the measured $K_s$ varied from $3.09 \times 10^{-3}$ to $3.36 \times 10^{-4}$ cm s$^{-1}$. The consolidated specimen tended to have the greatest $K_s$ value, followed by those prepared by Proctor compaction and static compaction. The observed difference highlights the importance of pore structure in determining $K_s$. This study also presents a pore-information-based theoretical approach for predicting $K_s$. A comparison of measured data shows that the proposed model performs better than the traditional void-ratio-based models.

Abbreviations: FID, free induction decay; K–C, Kozeny–Carman; NMR, nuclear magnetic resonance.

Saturated hydraulic conductivity ($K_s$) is a key physical variable of soil and is used to determine infiltration rate, percolation depth, and other hydrological processes (Malusis et al., 2003; Zhang et al., 2016; Jafari et al., 2017; Teng et al., 2019a). It has been revealed that $K_s$ depends on several factors such as soil texture and structure, chemical properties of the fluid, and pore structure (Hillel, 1982; Boynton and Daniel, 1985; Costa, 2006; Jang et al., 2011). A number of studies have investigated the effects of different sampling procedures on the anisotropy of $K_s$, which is caused by the anisotropy of pore structure and the differences in the cross-section of water flow (Bathke and Cassel, 1991; Burger and Belitz, 1997; Surridge et al., 2005; Iwanek, 2008; Bagarello et al., 2009). It is noted that the measurement of $K_s$ can be divided into in situ measurement and laboratory measurement. Some useful methods are widely applied to the in situ measurement of $K_s$—for example, the borehole measurements, tracer tests, and core sampling methods in combination with laboratory permeameter tests (Bouma and Dekker, 1981; Reynolds and Elrick, 1985; Amoozegar, 1989; Reeves et al., 1996; Beckwith et al., 2003; Germer and Braun, 2015). In the laboratory, $K_s$ can be measured by a constant head permeability test or the falling head permeability test. It is noted that the measured $K_s$ of the remolded soil is generally based on a compacted specimen. Such a compacted specimen is usually prepared by oven drying and sieving to dry soil powder, mixing with sprayed water to achieve a target initial gravimetric water content, and then static or dynamic compaction to a target dry unit weight. However, compacted specimens cannot represent the hydraulic properties of natural soil, particularly those consolidated from slurry deposits, or those residual soils from weathering (Reynolds 2008; Li and Zhang, 2009; Gao et al., 2016; Teng et al., 2016). The consolidated specimen is prepared by gradually increasing the loading pressure to an initially saturated soil until the soil volume reaches a target value. In these cases,
a consolidated specimen may represent the properties of natural soils more accurately.

The specimen preparation method has an evident influence on the pore size distribution of the soil (Delage et al., 1996). It has been reported that two populations of soil pores, interaggregate pores and intra-aggregate pores, are formed during the compaction of fine-grained soils (Li and Zhang, 2009). The interaggregate pores, which are not as stable as the smaller intra-aggregate pores, can be removed by either wetting or applying large external loading (Tarantino and de Col, 2008; Tarantino, 2011; Sheng et al., 2014).

Compared with compacted specimens, consolidated specimens tend to show a unimodal pore size distribution with a single population of soil pores (Gao et al., 2016). Considering that compaction is still the dominant sampling method in laboratory testing, questions are raised: for example, does specimen preparation influence $K_s$ or not, and how do you quantitatively evaluate the difference if it exists?

An accurate and reliable prediction of $K_s$ has been a long-standing topic of interest for geotechnical and geological researchers (Ren et al., 2016; Teng et al., 2019b). One of the most common models for $K_s$ is the Kozeny–Carman equation (referred to as the K–C equation):

$$K_s = C_F \left( \frac{1}{S_m^3} \right) \left( \frac{\gamma_w}{\mu + \mu_m} \right) \left( \frac{e^3}{1 + e} \right)$$  \[1\]

The K–C equation shows that $K_s$ (cm s$^{-1}$ or m s$^{-1}$) is governed by void ratio $e$ (unitless), soil density $\rho_m$ (kg m$^{-3}$), specific surface area $S_m$, which is defined as the total surface area of soil particle per unit of mass (m$^2$ g$^{-1}$), and a dimensionless shape constant $C_F$ if information on the fluid, such as the unit weight of the fluid $\gamma_w$ (N m$^{-3}$) and fluid viscosity $\mu$ (m$^2$ s$^{-1}$) (Kozeny, 1927; Carman, 1937), is known. Predicted results of the K–C equation agree well with experimental results for coarse-grained soils such as sand, but less so for fine-grained soils, largely because this equation neglects the electrochemical reaction between the solid particles and fluid (Carrier, 2003). Following the approach of the K–C equation, many researchers have suggested alternative relations for a wider range of soils. These works either modify the definitions of the parameters, such as the void ratio and specific surface area, in the K–C equation, or introduce a new parameter, such as the plastic limit or the diameter $D_{10}$ (10% of the particles is smaller than this diameter), to account for the effects of fine-grained particles (Taylor, 1948, p. 97–123; Wyllie et al., 1958; Carrier and Beckman, 1984; Dolinar, 2009; Indraratna et al., 2012; Kucza and Ilek, 2016; Ren and Santamarina, 2018). It is noted that most predictions of $K_s$ in the literature are based on information of the solid particles instead of information of the soil pores. In theory, $K_s$ depends more on the pore sizes and on how the pores are distributed and interconnected (Chapuis, 2012; Teng et al., 2014). A theoretical description of the complex pore structure of soil is needed to simulate $K_s$, although obtaining such a description may be challenging.

The objective of this study was to investigate the influence of specimen preparations on pore size distribution of soil and hence on $K_s$. The pore size distribution was measured by nuclear magnetic resonance (NMR) technology, which established a connection between the specimen preparations and $K_s$. In addition, a soil-pore-information-based mathematical model was developed to describe $K_s$.

**Materials and Methods**

**Laboratory Measurement**

The specimen used in the laboratory test was sampled at the depth of 2.0 m in an exploratory trench by cutting ring method. The sampling spot was close to the Zhongchuan Airport of Lanzhou City, China. The soil belonged to the Malan loess, which is composed of a kind of silty soil with low content of organic matter. The soil is a silty clay loam according to the International Soil Science Society (ISSS) classification, with 14% sand, 67% silt, and 19% clay, a specific gravity of 2.70 g cm$^{-3}$, liquid and plastic limits of 29.92 and 15.99%, an optimum moisture content of 17.00%, and a maximum dry density of 1.80 g cm$^{-3}$. The $D_{10}$ and $D_{60}$ of the specimen were 3 and 42 μm, respectively. The oven-dried sample was first placed on a flat plate surface, and then a certain amount of distilled water was sprayed evenly on the soil surface. A layer of dry soil was sprinkled on the wet surface, and the above process was repeated. The soil was then stirred and placed in a plastic bag for 24 h to ensure that the moisture in the soil had distributed uniformly. Therefore, a soil with a controlled initial water content of 17.00% was prepared for either Proctor compaction testing or static compaction testing.

The Proctor compaction test was conducted according to the ASTM D1557 (ASTM, 2009) standard. The dry density of the specimen was determined by controlling the compaction effort. For the static load compaction, a certain amount of soil was packed into the Proctor mold in two layers. A constant rate of axial loading pressure was applied to the specimen until the target density was achieved (Venkatarama Reddy and Jagadish, 1993; Islam and Kodikara, 2016). When preparing the consolidated specimen, the sample was mixed with a certain amount of distilled water, resulting in a slurry sample at two times the liquid limit. The slurry sample was decanted into a steal cylinder, and the consolidation pressure was applied step by step until the target height of the specimen was achieved. The final water content of the consolidated specimen was ~26%. Notably, the state conditions (void ratio, stress path, etc.) were strictly controlled to be the same for specimens with different specimen preparation methods, to ensure the validity of the comparison.

Three dry densities (1.40, 1.60, and 1.80 g cm$^{-3}$) were evaluated for the specimens in the Proctor compaction and static compaction tests. Two dry densities (1.40 and 1.60 g cm$^{-3}$) were used in the consolidation test. For each density, four specimens were prepared: two of them were used to measure the pore size distribution, and the other two specimens were used in the permeability test. Parallel testing of two specimens was designed to reduce the error of the experiment.
The specimens used for the permeability tests were obtained by using a 61-mm-i.d. cutting tube (40 mm in height). The $K_s$ was measured for each condition with a falling head method (ASTM D5084-10; ASTM, 2014). The pore size distribution of each specimen was measured twice by using the NMR technology. A photo of the NMR device is shown in Fig. 1. The NMR technology can evaluate the pore water content or pore size distribution by measuring the free induction decay (FID) curve. The $T_2$ relaxation times of protons in the porous medium can be obtained from the FID curve by applying the Fourier transformation. In water-saturated porous media, $T_2$ is linearly proportional to the pore size, such that the pore size distribution of the sample can be determined (Coates et al., 1999; Jaeger et al., 2009). It is noted that all the specimens were saturated under vacuum conditions before the permeability test and NMR test, to ensure that the conditions of soil pore structure during the two tests were consistent.

**Prediction of Saturated Hydraulic Conductivity Based on Pore Size Distribution**

In this section, a new model that takes into account the pore size distribution is introduced for predicting $K_s$. According to the capillary model, the pore channels in a soil are replaced by parallel capillary tubes (as shown in Fig. 2) (Deng et al., 2011; Ilek and Kucza, 2014). The ratio and the cross-sectional area of the capillary tubes are defined as $R$ and $a$, respectively. The head loss is defined as $h$ for a given length $L$, and the hydraulic gradient is $i = h/L$. The total seepage force of the tubular water body with radius $r$ is $\pi r^2 \gamma_w \rho$, where $\gamma_w$ is the unit weight of the soil. The surrounding water resistance is $2 \pi r \tau$, where $\tau$ is the water viscosity and is equal to $-\mu \nabla v/dr$, with $\mu$ and $v$ representing the dynamic viscosity coefficient and the flowing velocity, respectively.

We let the resistance equal the total seepage force, which can be expressed as

$$2\mu \nabla v = \gamma_w \rho \frac{dr}{d\tau}$$  \[2\]

The flow velocity through radius $r$ can be obtained by integrating Eq. [2] with the boundary conditions $r = R$ and $v = 0$:

$$v = \frac{\gamma_w}{4\mu} \left( R^2 - r^2 \right)$$  \[3\]

The flow through cross-section $R$ is

$$Q = \int_0^R v 2\pi r dr$$  \[4\]


$$Q = \frac{\pi R^4 \gamma_w}{8\mu} = \frac{R^2 \gamma_w}{8\mu} \cdot i \cdot a$$  \[5\]

Substituting $Q = K_s A$ and $a = n A$ into Eq. [5] results in

$$K_s = \frac{n R^2 \gamma_w}{8\mu}$$  \[6\]

where $n$ is the porosity of the soil. In Eq. [6], the key parameter for determining $K_s$ is the representative pore radius $R$. The weighted average method or the mean value of the associated soil pores were commonly used to represent the pore size $R$. However, such an average value cannot accurately represent the pore structure of soil, as it neglects the interaction among the pores with different sizes (Leonards, 1962; Garcia-Bengochea et al., 1979). Here, the pore-throat effect is introduced to account for the restriction of liquid water passing from large pores into small pores (Marshall, 1958, 1959). Marshall (1958) proposed an estimation method for the representative pore radius $R$:

$$R^2 = \frac{n r_1^2 + 3r_2^2 + 5r_3^2 + \ldots + (2m - 1) r_m^2}{m^2}$$  \[7\]

where the mean radius of the pores in each of $m$ equal fractions of the total pore space is represented in decreasing order of size by $r_1$, $r_2$, $r_3$, ..., and $r_m$ (cm), respectively. An inherent assumption exists in the derivation of Eq. [7]: all the soil pore sizes have the same probability distribution. However, in real soils, the soil pore distribution exhibits a normal probability function. A new
The pore radius of \( r_1 \) is also affected by the cross-section of the pore radius \( r_2 \). The cross-sectional area related to \( r_2 \) can thus be expressed as:

\[
\pi (2r_2^2 + ... + r_m^2)/m,
\]

leading to the total cross-sectional area of water volume that must be considered being:

\[
\pi [(r_1^2 + r_2^2 + ... + r_m^2) + (2r_2^2 + ... + r_m^2) + ... + mr_m^2] = \pi (r_1^2 + 3r_2^2 + 5r_3^2 + ... + (m - 1)r_m^2)/m.
\]

Furthermore, the probability distribution function of the pores should be taken into account, since the number of soil pores differ with pore size. The corresponding probabilities for pore sizes \( r_1, r_2, ..., r_m \) are expressed as \( \omega_1, \omega_2, ..., \omega_m \). The representative pore radius \( R \) can then be written as:

\[
R^2 = \frac{\omega_1 r_1^2 + \omega_2 3r_2^2 + ... + \omega_m (2m - 1)r_m^2}{m}
\]  

where \( \omega \) can be determined according to the pore size distribution curve. Substituting Eq. [8] into Eq. [6] leads to the new method for computing the hydraulic conductivity.

It is noted that a number of expressions have been suggested in the literature for estimating the saturated hydraulic conductivity of soils. Ren et al. (2016) made contrastive analysis for these formulas and proposed a new model for \( K_s \) by introducing a new concept of effective void ratio. The result in Ren et al. (2016) shows that the proposed model has reasonable performance for a wide range of soils. The expression is as follows:

\[
K_s = C \frac{\epsilon_t^{3b+3}}{(1 + \epsilon_t)^{3/2}b+1 \left[ (1 + \epsilon_t)^{b+1} - \epsilon_t^{b+1} \right]^{1/3}}
\]  

where \( b \) is a positive constant for a given soil; \( \epsilon_t \) is the total void ratio, which is the summation of the effective void ratio and the ineffective void ratio; and the parameter \( C \) is equal to \( \gamma_w/C_D\mu \rho_m^2S_0^2 \). Therefore, there are some existing methods that can be used to compare with the new model, including the K–C equation (Eq. [1]), Marshall (1958)'s model (Eq. [6] and [7]), and the model of Ren et al. (2016) (Eq. [9]).

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**Results and Discussion**

**Effect of Specimen Preparation on Saturated Hydraulic Conductivity**

Figure 3 presents the relationship between \( K_s \) and dry density for different specimen preparations. It shows that a greater dry density led to a smaller \( K_s \), irrespective of the sample preparation methods. The value of \( K_s \) at the density of 1.4 g cm\(^{-3}\) was approximately five to six times greater than \( K_s \) at 1.8 g cm\(^{-3}\). The result also indicates that \( K_s \) is significantly influenced by the sample preparation method. For a given dry density, such as 1.6 g cm\(^{-3}\), \( K_s \) for the consolidated specimen was greater than that for Proctor compaction, whereas static compaction generated the lowest value of \( K_s \). The differences among these three specimen preparation methods can be as large as one order of magnitude, which implies that \( K_s \) varies significantly even if the particle size and dry density (void ratio) of the specimens are kept the same. Therefore, it is challenging to determine the hydraulic conductivity accurately using only the information of solid particles. In theory, \( K_s \) is related to the pore structure, but the pore information cannot easily be obtained from solid particles.

Figure 4 shows the effect of specimen preparation on pore size distribution under different dry densities. The pore volume ratio is defined as the total volume of soil pores divided by the volume of a certain pore size, which is determined by the signal intensity of the liquid water in a saturated soil specimen by the NMR technology. This figure shows that the specimen prepared by static compaction had very few soil pores >10 \( \mu \)m, unlike the other two preparation methods, leading to lowest saturated hydraulic conductivity. Because the large pores were mainly interaggregate pores, they could provide early flow conduits for the liquid water. Comparing the pore size distribution of the Proctor compaction and consolidated specimens, as shown in Fig. 4b and 4c, the pore size of the consolidated specimen mainly concentrated in the range of 1.6 to 6.3 \( \mu \)m, which presents a more centered pore size distribution. Because the hydraulic process in the maximal pore is controlled by its surrounding smaller capillary pores and the cross-sectional area is controlled by the mean value of all the pore sizes (Marshall, 1958), a more centered pore size distribution indicates a weaker pore throat, leading to a greater hydraulic conductivity.

To verify the new model, the K–C equation and the models proposed by Marshall (1958) and Ren et al. (2016) were used to...
predict the values of $K_s$. All the predicted results were compared with the measured data. The inputs of these models are summarized in Table 1, and the computed and measured data of $K_s$ are presented in Fig. 5.

It is noted that the K–C equation and Ren et al.’s (2016) model, which are both derived from the void ratio, can only generate a single result for one dry density. Marshall’s (1958) model computes $K_s$ based on the soil pore size distribution. It can present three groups of results, as shown by the dashed lines in Fig. 4. The results show the proposed method has a better prediction performance than the other methods. The predictions of Marshall’s (1958) model are approximately two to three times greater than the measured values. Compared with the traditional methods in the literature, the proposed model can reveal the essence of seepage flow in soil and generate accurate $K_s$ values. Its cost is that the pore size distribution must be known a priori.

![Fig. 4. Pore volume ratio vs. pore radius for different specimen preparations. The dry densities are (a) 1.40, (b) 1.60, and (c) 1.80 g cm$^{-3}$.](image)

![Fig. 5. The comparison between the computed and measured results of saturated hydraulic conductivity ($K_s$). The solid lines indicate the results predicted by the proposed model: the Marshall (1958) model, the Ren et al. (2016) model, and the Kozeny–Carman (K–C) equation.](image)

<table>
<thead>
<tr>
<th>Models</th>
<th>Density 1.40 g cm$^{-3}$</th>
<th>Density 1.60 g cm$^{-3}$</th>
<th>Density 1.80 g cm$^{-3}$</th>
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<tbody>
<tr>
<td>Kozeny–Carman equation</td>
<td>$C_F = 0.20$; $\mu = 1.01$; $\rho_m = 1.40$; $S_s = 11.28$; $\epsilon = 0.93$; $\gamma_w = 9.80$; $g_w = 9.80$</td>
<td>$C_F = 0.20$; $\mu = 1.01$; $\rho_m = 1.60$; $S_s = 11.28$; $\epsilon = 0.69$; $\gamma_w = 9.80$; $g_w = 9.80$</td>
<td>$C_F = 0.20$; $\mu = 1.01$; $\rho_m = 1.80$; $S_s = 11.28$; $\epsilon = 0.50$; $\gamma_w = 9.80$; $g_w = 9.80$</td>
</tr>
<tr>
<td>Ren et al. (2016) model</td>
<td>$e_t = 0.93$; $b = 1.10$</td>
<td>$e_t = 0.69$; $b = 1.10$</td>
<td>$e_t = 0.50$; $b = 1.10$</td>
</tr>
<tr>
<td>Marshall (1958) model</td>
<td>measured data of the pore size distribution as shown in Fig. 4</td>
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† $C_F$, a dimensionless shape constant; $\mu$, fluid viscosity; $\rho_m$, soil density; $S_s$, specific surface area; $\epsilon$, void ratio; $\gamma_w$, unit weight of the fluid; $b$, positive constant for a given soil; $e_t$, total void ratio.

‡ The values of $C_F$, $\mu$, $\rho_m$, $S_s$, and $g_w$ in Ren et al. (2016) are the same as that of the Kozeny–Carman equation (Eq. [1]).
Conclusions

The hydraulic conductivity $K_s$ of soil is usually difficult to measure or predict. The influence of specimen preparations on $K_s$ has rarely been reported. In this study, laboratory tests and theoretical analysis were performed in attempt to gain new understanding of $K_s$. The following conclusions can be drawn based on the results.

1. Specimen preparation methods have a considerable influence on $K_s$ because they lead to the formation of different pore size distributions. Under the same dry density (or the same porosity), the consolidated specimen tends to have the largest $K_s$ value, followed by those from Proctor compaction and static compaction. The difference among these specimens can be as large as one order of magnitude, indicating that the pore size distribution should be considered in predicting $K_s$.

2. A pore-information-based theoretical model is presented for estimating $K_s$, taking into account the interaction between large and small pores and the soil pore size probability distribution. The comparison with the measured data shows that the proposed model is more accurate than traditional void ratio-based models in the literature.

It is noted that the pore size distribution of soil is assumed to be immobile during the permeability test. Further studies should pay attention to the change of soil pore structure when the water is flowing through soil pores. Nevertheless, this study provides new insight into understanding the hydraulic conductivity of soils. The measured results can be used to validate models of hydraulic conductivity.

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References


