Steady Water Flow with Interacting Point Source–Point Sink–Water Table in a Cylindrical Soil Domain

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A previously derived analytical solution to the quasi-linear form of the water flow equation is used to analyze (i) steady, coupled plant water uptake from a surface water emitter in a confined cylindrical soil domain with a non-evaporating surface in the presence of a shallow water table, and (ii) water uptake from only the water table in the absence of a surface emitter. Illustrative examples serve to analyze and discuss water-uptake rates of a subsurface, spherical, conceived root zone and the complex water-flow patterns occurring in either natural fields with shallow groundwater or artificial lysimeters. The coupled source–sink–water table model is also used to illustrate the dependence of the contributions of surface emitter and water table to the overall water-uptake rate on capillary length and hydraulic conductivity of the saturated soil, the depth of the water table and its prescribed pressure head, the depth and size of the root zone, and the radius of the confined cylinder, representing the effect of neighboring plants and emitters. The proposed methodology can be used to evaluate the effects of these factors on the potential utilization of shallow groundwater, as well as in cases with a supplementary drip irrigation system, and to support design decisions concerning the distance between emitters (and between plants) and the irrigation rates required to complement plant water uptake from groundwater.

Abbreviations: DB, dry bottom; ET, evapotranspiration; MFP, matric flux potential; RWUR, relative water-uptake rate; Si, sink; So, source; WT, water table; WUR, water-uptake rate.

Many lysimeter studies have evaluated the potential contribution and many field trials have explored the actual contribution of shallow groundwater to the water consumption and evapotranspiration (ET) of various annual and perennial crops, with or without precipitation or supplemental irrigation. For a comprehensive review of these studies, see Ayars et al. (2006). Taking into account precipitation or supplementary irrigation, the depth of the groundwater table and its quality (mainly salinity), the soil texture, the vertical distribution of the roots, irrigation regime, and climatic conditions, the contribution of shallow groundwater to overall water consumption by crops has been estimated at, for example, between 15 and 50% (Hoffman et al., 1990) or up to 42% for cotton (Gossypium hirsutum L.; Hutmacher et al., 1996); 15% (Wang et al., 2018), 18% (Babajimopoulos et al., 2007), or 60% for corn (Zea mays L.; Logsdon et al., 2009); 59% for safflower (Carthamus tinctorius L.; Ghamarnia and Gholamian, 2013); 87% for soybean [Glycine max (L.) Merr.; Logsdon et al., 2009]; 92% for potato (Solanum tuberosum L.; Satchithanantham et al., 2014); and 100% for alfalfa (Medicago sativa L.; Ayars et al., 2009). The contribution of the water table (WT) to water uptake is naturally higher the shallower it is and the fewer alternative, on-surface water sources there are. For example, in a lysimeter filled with a fine sandy loam subjected to two irrigation regimes, three WTs controlled at the 91-, 183-, and 274-cm depths contributed 54.4, 26.4, and 17.3%, respectively, of the total water use by cotton under the wetter treatment and 60.6, 48.9, and 39.2%, respectively, under the drier treatment (Namken et al., 1969). In another field study, cotton grown in a loam soil received about 64% of its ET demand from a 212- to 266-cm-deep WT, and the lower the number of irrigations, the higher the groundwater’s contribution to ET (Wallender et al., 1979). For a given WT depth, plants with deeper roots are expected to better exploit the.
groundwater (Ayars et al., 2006). Ayars et al. (2006) also claimed that for a given WT depth, the percentage of water taken up from the groundwater decreases with increasing soil clay content. However, based on soil water flow considerations (disregarding, e.g., soil salinity and aeration aspects), it is expected that for each WT depth, there is a soil texture that results in maximum groundwater uptake, and that the deeper the WT, the more clayey the soil that provides the maximum water uptake.

Predictions of shallow groundwater contribution to the water consumption of various crops are either empirical (Ragab and Amer, 1986; Grismer and Gates, 1988; Prarthapar and Meyer, 1993; Sepaskhah et al., 2003) or based on numerical simulations of saturated–unsaturated, one-dimensional water flow and uptake (Zhang et al., 1999; Chen and Hu, 2004; Zhu et al., 2009; Shouse et al., 2011; Karimov et al., 2014).

Surface and subsurface point source emitters are widely used for irrigation. Their water flow patterns have been extensively studied and documented, and they can be reliably reproduced with software packages such as HYDRUS (Šimůnek et al., 2014) or DIDAS (Drip Irrigation Design and Scheduling: Communar et al., 2015; Friedman et al., 2016). The presence of a WT at shallow depth can greatly affect the flow patterns in the overlying soil domain, as addressed, for example, by the analytical studies of Warrick and Lomen (1977), Philip (1989), Martinez and McTigue (1991), de Rooij et al. (1996), and Basha (2000). Communar and Friedman (2015) derived an analytical solution for water flow from (or to) a surface (or subsurface) point source (or sink) in a confined cylindrical domain with a lower boundary consisting of a shallow WT of a prescribed pressure head and an upper boundary consisting of an evaporating soil surface. They also demonstrated the effects of WT depth, cylinder radius, and evaporation on flow patterns that arise under the action of either a single point source or point sink. The main objectives of the current study were to apply this solution in a coupled source–sink water-uptake model and propose this basic analytical tool, to evaluate the relative contributions of both drip irrigation and groundwater to the plants’ water uptake in the presence of shallow groundwater, and to discuss the effects of WT depth, root zone depth, soil texture, and the distance between plants (and between emitters, when applicable) on these contributions in the presence or absence of supplementary drip irrigation.

The analytical solution for the point source applied to the analysis (Communar and Friedman, 2015) uses Gardner’s (1958) model, in which hydraulic conductivity is an exponential function of the pressure (matric) head, which enables linearization of Richards’ equation for steady flow. In Communar and Friedman (2015), a general form was derived and presented that also accounts for evaporation from the soil surface and vertical variation of the soil hydraulic conductivity at saturation. In the following, it is presented in a reduced form for the case of no vertical flux at the soil surface and for a homogeneous soil profile. The description of water uptake follows the coupled source–sink approach proposed previously to describe water flow and uptake in semi-infinite soil domains without shallow groundwater (Communar and Friedman, 2010, 2011), implemented in the DIDAS software.

Theory

A Point Source in a Finite Cylinder

Following Communar and Friedman (2015), consider a point source of strength $q_{so} \left[ \text{L}^3 \text{T}^{-1} \right]$ at $r = 0, z = z_{so}$. The soil volume is assumed to be bounded by $r = r_c$ by an impermeable cylindrical wall and at $z = z_0$ by a horizontal plane with a constant, non-positive pressure head $b_0 \leq 0$ (Fig. 1, depicting the special case of a water source at the soil surface, $z = 0$; the origin of the coordinates is at the soil surface and the $z$ axis is positive in the downward direction). For a free WT, $b_0 = 0$ and a negative value of $b_0 = -\infty$ corresponding to the maximum possible (theoretically infinite) suction is assumed for a dry-bottom, cylindrical lysimeter.

For a soil whose hydraulic conductivity $K$ varies exponentially with the pressure (matric) head, $h$ [K = $K_1 \exp(\alpha h)$, where $K_1$ is the hydraulic conductivity at water saturation and $\alpha^{-1}$ is the soil capillary length], it is constructive to refer to the matric flux potential ($\varphi$) [L$^2$ T$^{-1}$]:

$$\varphi = \int_{-\infty}^{b} K(h) db = \frac{K_c \exp(\alpha b)}{\alpha}$$

![Fig. 1. Schematic description of the flow domain and the coupled water source–root sink model. The boundary condition (b.c.) on the vertical cylindrical wall ($r = r_c$) is of zero radial water flux and that on the soil surface is of zero vertical water flux. The boundary condition at the lower surface ($z = z_0$) is of a prescribed, non-positive pressure head ($-\infty \leq b_0 \leq 0$). A steady point source of strength $q_{so}$ is located on the surface (and at the origin of the cylindrical coordinate system, 0,0) of a soil of capillary length $\alpha^{-1}$. The point sink of (sought-after) strength $q_{si}$ is located at 0, $z_{si}; z_{si} (= r_0)$ in the illustrative examples), and $r_0$ is the radius of the conceived (spherical) root zone, describing the soil volume outside of which there are no active roots. To evaluate the contributions of both the surface point source and the water table to the water-uptake rate, a condition of maximum possible suction ($h_{si} = 0$) is applied at the reference point at the bottom of the conceived root zone domain (0, $z_{si} + r_0$), and the contour of $\varphi_{si} = 0$ (schematically, the surface of the shaded ellipsoid) delineates the actual sink, which is the soil volume exhibiting maximum suction.](image-url)
Normalizing all physical lengths with respect to 2α−1 (Philip, 1989).

\[
\frac{R}{r} = \frac{Z}{z} = \frac{R_c}{z_0} = \frac{Z_{so}}{z_{so}} = \frac{\alpha}{2}
\]  \hspace{1cm} [2]

and using the dimensionless MFP created by the point source, \( \Phi_{so} = 8\pi \varphi / \alpha^2 \), the appropriate form of the dimensionless, linearized steady flow equation in cylindrical coordinates is

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi}{\partial R} \right) + \frac{\partial^2 \Phi}{\partial Z^2} - 2 \frac{\partial \Phi}{\partial Z} = -\frac{2}{R} \delta(Z - Z_{so})
\]  \hspace{1cm} [3]

where \( (2/R) \delta(Z - Z_{so}) \) is a localized point source, expressed with the Dirac delta function \( \delta(\cdot) \) defining its location at \( R = 0, Z = Z_{so} \) \((0 < Z_{so} < Z_0)\). The boundary conditions with respect to the \( R \) axis are \( \partial \Phi / \partial R = 0 \) for \( R = 0 \) and \( R = R_c \), and with respect to the \( Z \) axis they are no-flux, \( -\partial \Phi / \partial Z + 2\Phi = 0 \), at the soil surface \( (Z = 0) \) and a constant value of \( \Phi = \Phi_{so} \) at \( Z = Z_0 \) \((or in its dimensional form: \( \Phi_0 \) = \( K / \alpha^2 \) \exp(\alpha b_0))\). The equilibrium (hydrostatic) distribution of the MFP that arises due to a non-zero value of \( \Phi_{so} \) at the lower boundary \( Z = Z_0, \Phi_{wi}(Z) \), corresponds to the onedimensional solution of the steady-state Eq. [3] with no point source term:

\[
\Phi_{wi}(Z) = \phi_{so} \exp[-2(Z - Z_{so})]
\]  \hspace{1cm} [4]

and the overall MFP due to both the point source and the WT is

\[
\Phi(R, Z) = \phi_{so} + \Phi_{so}(R, Z)
\]  \hspace{1cm} [5]

where \( \Phi_{so}(R, Z) \) is the solution to Eq. [3] for \( \phi = 0 \) at \( Z = Z_0 \) derived by Communard and Friedman (2015):

\[
\Phi_{so}(R, Z) = \frac{2}{R_c^2} \Phi_{hi}(\mu_{so}, Z) + \frac{2}{R_c^2}  \sum_{n=1}^{\infty} \frac{J_0(\sigma_n R_c)}{J_1(\sigma_n R_c)} J_1(\sigma_n R_c) \Phi_{hi}(\mu_n, Z)
\]  \hspace{1cm} [6]

where \( J_n \) is the zero-order Bessel function of the first kind and \( \sigma_n \) denotes the \( n \)th zeros of the first-order Bessel function \( J_1(\sigma_n R_c) = 0 \) of the first kind, \( \omega_n = (1 + \mu_n^2)^{1/2} \), \( \mu_n = \lambda / Z_0 \) is the \( n \)th positive root of the characteristic equation \( \lambda \cot(\lambda) + Z_0 = 0 \), and

\[
\Phi_{hi} = \frac{\exp(Z - Z_{so})}{\omega_n} \{ \exp[-\omega_n(Z - Z_{so})] - \exp[\omega_n(Z - Z_{so})] \}
\]  \hspace{1cm} [7]

\[
\Phi_{hi} = 2 \exp(Z - Z_{so}) \frac{D(\omega_n, Z) \sinh[\omega_n(Z - Z_{so})]}{\omega_n D(\omega_n, Z)}
\]  \hspace{1cm} [8]

with \( D(\omega_n, Z) \) defined by

\[
D(\omega_n, Z) = (\omega_n + 1) \exp(\omega_n Z) + (\omega_n - 1) \exp(-\omega_n Z)
\]  \hspace{1cm} [9]

The normalized stream function, \( \Psi = \psi / q_{so} \), for the point source is

\[
\Psi_{so}(R, Z) = \frac{R^2}{R_c^2} G(Z) + \sum_{n=1}^{\infty} \frac{R}{R_c} \frac{J_1(\sigma_n R_c)}{(\sigma_n R_c) J_0(\sigma_n R_c)} \frac{G(\omega_n, Z)}{\omega_n D(\omega_n, Z)}
\]  \hspace{1cm} [10]

where

\[
G(\omega_n, Z) = \frac{\exp(Z - Z_{so})}{2\omega_n} \{ (1 - \omega_n) \exp[-\omega_n(Z - Z_{so})] - (1 - \omega_n) \exp[\omega_n(Z - Z_{so})] \}
\]  \hspace{1cm} [11]

The procedures for computing the various terms appearing in \( \Phi_{so}(R, Z) \) and \( \Psi_{so}(R, Z) \) were described by Communard and Friedman (2015).

Here, we refer exclusively to a surface point source (Fig. 1, \( \Phi_0 = 0 \)) and a subsurface point sink of negative strength \( q_{si} \), located at \( R = 0, Z = Z_{si} \), whose MFP \( \Phi_{si}(R, Z) \) and stream function \( \Psi_{si}(R, Z) \) are given by the same Eq. [6–8] and [9–11], respectively. Note that for \( \Phi_0 > 0 \), a physically relevant solution to \( \Phi \) by Eq. [3] exists not only for scenarios with a point source but also for solely a point sink \( \Phi_{si}(R, Z) \), which extracts water at a limited rate, \( q_{si} \). But for \( \Phi_0 = 0 \) (no water supply from a WT), a solution to Eq. [3] exists only for a source function.

**Coupled Point Source–Sink in the Presence of a Dry Soil Lower Boundary**

Consider a surface point source of strength \( q_{so} \) and a point sink that is located at a prescribed depth below the source \( (\varepsilon_{si}) \) and extracts water from a soil at a sought-after rate of \( q_{si} \) (Fig. 1). The evaluation of the water uptake rate \( (WUR) \), \( q_{si} \), follows the coupled source–sink approach proposed previously for describing water flow and uptake in semi-infinite soil domains without shallow groundwater (Communard and Friedman, 2010, 2011). To evaluate the sink WUR contributed by the surface emitter, \( q_{si so} \), a completely dry bottom of the active root zone is assumed, i.e., \( \varphi = 0 \) or \( h_0 = -\infty \) (choosing another, negative enough, matric head value does not much affect the evaluated water uptake rate; Communard and Friedman, 2010). Thus, the sink-from-surface-emitter strength \( (q_{so ref}) \) is determined by applying the condition \( \varphi(0, Z_{ref}) = 0 \) to the bottom of the sink zone, \( Z_{ref} = Z_{si} + R_0 \), where \( R_0 = \alpha r_0 / 2 \) denotes the radius of the spherical, conceived root zone (Fig. 1), and it can be expressed as

\[
q_{so ref} = \frac{\Phi_{so}(0, Z_{ref})}{\Phi_{so}(0, Z_{ref})}
\]  \hspace{1cm} [12]

The potential \( \varphi \) for the flow induced by the surface point source and the subsurface point sink is obtained by superposition of the appropriate solutions for \( \Phi_{so} \) and \( \Phi_{si} \), respectively, and is given by

\[
\phi(R, Z) = \frac{\alpha q_{so}}{8\pi} \Phi_{so}(R, Z) - \frac{\alpha q_{si ref}}{8\pi} \Phi_{si}(R, Z)
\]  \hspace{1cm} [13]
Notice that a soil volume with (non-physical) negative values of \(\varphi\) around the sink is inscribed by the \(\varphi = 0\) iso-MFP surface (gray-colored, ellipsoidal actual sink in Fig. 1). Equations [12] and [13] apply for both semi-infinite and finite cylindrical soil domains. It is assumed that in a semi-infinite cylinder, the conditions \(R_0 \leq Z_{si}\) and \(R_0 \leq R_c\) are satisfied and that in a finite-length cylinder, the restriction \(Z_{ref} < Z_0\) is also satisfied. For given source and sink locations, the relative water-uptake rate (RWUR \(= q_{si\_so}/q_{so}\)) of the sink uptake rate relative to the point source flow rate) in a semi-infinite cylinder depends on \(R_c\) and \(R_0\), and in a finite-length cylinder, it also depends on the distance between \(Z_{ref}\) and \(Z_0\).

**Coupled Point Sink–Water Table**

The description of this process was formulated in detail by Communar and Friedman (2015) and is briefly repeated here. In the absence of a surface emitter, the point sink, located at a depth \(Z = Z_{si}\), extracts water from a soil that is otherwise under hydrostatic conditions (equilibrium MFP--\(b\)--water content distribution, Eq. [4]), and the \(\varphi(R,b)\) distribution is determined by Eq. [3], replacing \(q_{so}\) with \(q_{si\_wt}\), the unknown, sought-after strength of the WT contribution to the sink rate, and \(\Phi_{si}\) by \(8\pi \varphi(0,0) q_{si\_wt}\). For consistency, the location of the reference point is chosen at the same location at the bottom of the root zone, and applying the condition \(\varphi(0,Z_{ref}) = 0\) to a modified Eq. [5] gives

\[
q_{si\_wt} = \frac{8\pi \varphi(0) \exp[-2(Z_0 - Z_{ref})]}{\Phi_{si}(0, R_0)} \tag{14}
\]

In addition to the above-listed geometrical attributes, \(q_{si\_wt}\) depends on the prescribed pressure head at \(Z_0, h_0\), and the soil’s hydraulic conductivity at saturation, \(K_s\).

**Coupled Point Source–Point Sink–Water Table Problem**

The general problem analyzed here is of an interacting surface point source–subsurface point sink–WT system and in particular the evaluation of the two contributions—the point source (on-surface emitter) and the WT—to the overall water uptake of the subsurface point sink (root system). Due to the linearity (in MFP, \(\varphi\) of the steady water flow equation (Eq. [3]), the overall problem of point source–point sink–WT in a laterally confined cylinder (source [So]-sink [Si]-WT, left-hand cylinder in Fig. 2) can be decoupled into two separate and independent problems: (i) a coupled point source–point sink in a confined cylinder with a totally dry lower boundary (So-Si-so dry bottom [DB], middle cylinder) and (ii) a point sink–WT (Si-WT, right-hand cylinder). Since the boundary condition on the lower boundary of the finite cylinder is of constant MFP (or pressure, usually termed a first-kind boundary condition), the MFP at the lower boundary of the combined So-Si-WT scenario (left-hand cylinder) is the sum of the two constant potentials of the two component problems: a zero MFP (\(\varphi = 0\)) in the So-Si-DB problem (middle cylinder) and a positive, less than the saturated value MFP (0 < \(\varphi \leq K_s/\alpha\)) in the Si-WT problem (right-hand cylinder). The boundary condition on the upper plane of the finite cylinder, i.e., on the soil surface, is the water flux (usually termed a third-kind boundary condition for the present flow equation, Eq. [3]), assumed zero (no evaporation) here, and it is identical in all three problems. The MFP of the combined scenario is the sum of the MFPs of the two decoupled problems:

\[
\Phi_{si\_so\_wt}(R,Z) = \Phi_{si\_so\_indep}(R,Z) + \Phi_{si\_wt}(R,Z) \tag{15}
\]

and similarly, the overall WUR of the sink (\(q_{si}\)) is also the sum of the two independently evaluated contributions of the WT (\(q_{si\_wt\_indep}\)) and the point source (emitter) (\(q_{si\_so\_indep}\)):

\[
q_{si} = q_{si\_wt\_indep} + q_{si\_so\_indep} \tag{16}
\]

However, the respective contributions of the WT (\(q_{si\_wt\_indep}\)) and the surface point source (\(q_{si\_so\_indep}\)) to the overall WUR of the sink (\(q_{si}\)) in the combined So-Si-WT problem are different from their independently evaluated contributions in the Si-WT (\(q_{si\_wt\_indep}\)) and So-Si-DB (\(q_{si\_so\_indep}\)) problems. The contribution of the surface point source to the sink strength in the presence of the water table (\(q_{si\_so}\)) is larger than that evaluated independently (\(q_{si\_so\_indep}\)), whereas that of the water table in the presence of
the surface point source \( q_{si_{-wt}} \) is smaller than its independently evaluated counterpart \( q_{si_{-wt}} \). The upward flow from the WT interferes with the downward flow from the surface point source, therefore more flow from the point source is directed to the sink than for the independent case. In other words, the water velocity field in the combined So-Si-WT problem is a vector summation of the independent Si-WT and So-Si-DB velocity fields, and velocity vectors in opposite directions partially cancel each other out. When there is no deep percolation from the surface point source to the WT, \( q_{si_{-wt}} \) is evaluated by radially integrating the vertical upward flux \( q_{si_{-wt}} \) at the WT \( z_0 \), from the cylinder axis \( (r = 0) \) to its wall \( (r = r_c) \):

\[
q_{si_{-wt}} = -2\pi \int_0^{r_c} v_z (r, z_0) r dr
\]

or otherwise by integrating from the cylinder axis to the radial location of the stagnation point \( (r = r_{stagn}) \), which divides the flow field between upward flow toward the sink in the cylinder core and downward drainage in the periphery:

\[
q_{si_{-wt}} = -2\pi \int_0^{r_{stagn}} v_z (r, z_0) r dr
\]

The contribution of the point source to the overall WUR \( q_{so} \) is then calculated by subtracting the newly evaluated contribution of the WT to the overall WUR from the overall WUR:

\[
q_{si_{-so}} = q_{so} - q_{si_{-wt}}
\]

The rate of deep percolation \( q_{dp} \) from the surface point source to the WT is evaluated from the difference between the point source strength and its contribution to the sink strength, and by virtue of a water volume balance it is also equal to the radially integrated vertical upward flux at the WT from the stagnation point to the cylinder wall:

\[
q_{dp} = q_{so} - q_{si_{-so}} = -2\pi \int_{r_{stagn}}^{r_c} v_z (r, z_0) dr
\]

This closes the balance of the flow rates, i.e., the net outflow rate from the WT in the overall So-Si-WT problem equals the deep percolation flux from the point source \( q_{dp} \), DP, left-hand cylinder, Fig. 2) minus the upward flux from the WT \( q_{si_{-wt}} \), lower arrow WU, left-hand cylinder, Fig. 2).

**Illustrative Examples and Discussion**

Communar and Friedman (2015) demonstrated the effects of WT depth, cylinder radius, and evaporation on flow patterns that arise under the action of either a single point source or a point sink in the presence of a WT. In the following, we first analyze the two decoupled problems, So-Si-DB and Si-WT, and then the combined So-Si-WT problem. We will illustrate the steady flow patterns arising from the action of a coupled point source–sink pair in the presence of a shallow WT (or a totally dry lower cylinder boundary) and the effects of the cylinder radius \( r_c \) and depth \( z_0 \), the radius of the conceived root zone \( r_{so} \), and the prescribed pressure head at the WT \( h_0 \) on the potential contributions of the point source (on-surface emitter) and of the WT to the overall WUR. The analytical solution to \( \varphi \) by Eq. [6] and the expression for the stream function by Eq. [9] are used in the following examples to compute the WUR and the spatial distributions of \( h \) \((= (1/\alpha) \ln(\alpha \varphi/K) ) \) and \( \Psi \) in finite-length cylinders \((0 \leq r \leq r_c, 0 \leq z \leq Z_0) \) in the presence of a WT \((\varphi_0 > 0, h_0 > -\infty) \). In the following, we use the terminology of dry bottom (DB) for the cases of \( h_0 = -\infty \), WT for \( -\infty < h_0 < 0 \), and free WT for \( h_0 = 0 \). Unless otherwise indicated, the illustrated examples refer to cylinders of a radius \( r_c = 25 \) or 50 cm and a depth of \( z_0 = 50 \) or 100 cm, using the values of \( \alpha = 0.04035 \text{ cm}^{-1} \) and \( K = 1 \text{ cm h}^{-1} \). The ratio between these two soil coefficients corresponds to an empirical equation, \( \alpha/K^{1/2} = \beta \), for which the coefficient \( \beta \approx 0.04035 \text{ cm}^{-3/2} h^{1/2} \) provides a reasonably good fit to the experimental data of \( \alpha \) and \( K \) (Communar and Friedman, 2010, 2014). The point sources (on-surface emitters) of strengths \( q_{so} = 0.5 \) or 1.0 L h\(^{-1} \) are located on the soil surface, and the point sinks are at depths of \( z_{si} = 10 \) or 20 cm below the soil surface.

**Water-Uptake Rates and Flow Fields in Finite and Semi-infinite Cylinders with a Dry Lower Boundary**

We first wish to evaluate the effects of the length \( z_0 \) of a finite cylinder, as well as its radius \( r_c \), on the WUR by a sink, assuming \( \varphi(z_0) = \varphi_0 = 0 \), i.e., a lysimeter with maximum possible suction at its bottom. Figure 3 shows plots of RWURs calculated with Eq. [12] for a conceived sink of radius \( r_s = 10 \) cm located at \( z_s = 10 \) cm below the soil surface in finite-length cylinders of radii \( r_c = 25 \) and 50 cm, whose depth \( z_0 \) varies from 20 to 100 cm. It is not strictly intuitive that the RWUR \( q_{si_{-so}}/q_{so} \) in a finite-length, DB cylinder is smaller than in an equivalent, semi-infinite cylindrical domain. Because the effect of the dry lower boundary on the nearer sink

![Fig. 3. Relative water uptake rate (RWUR, \( q_{si_{-so}}/q_{so} \)) for a coupled source and sink in finite and semi-infinite cylinders of radii \( r_c = 25 \) and 50 cm. The conceived sink is located at depth \( z_s = 10 \) cm below the source and has a radius of \( r_s = 10 \) cm. The RWUR is evaluated with a reference point located at the bottom of the conceived root zone, here and elsewhere, unless otherwise stated \( z_{ref_{so}} = 20 \) cm. Solid lines represent RWURs in finite cylinders with \( \varphi_0 = 0 \) \( (h_0 = -\infty) \) at depth \( z_0 \) (their bottom) varying from 20 to 100 cm, and dashed lines show the RWURs in semi-infinite cylinders \((\varphi = 0 \text{ at } z \to \infty) \) of the same radii.
is larger than its effect on the farther source, the nondimensional potential created by the sink at the reference point, \( \Phi_{s0}(0, Z_{ref}) \), is more enhanced by the dry lower boundary than is the nondimensional potential created by the source in that location, \( \Phi_{so}(0, Z_{ref}) \); this is why the RWUR \((q_{si}/q_{so})\) increases with increasing cylinder depth, \( z_0 \). The minimum values of the RWUR, 0.47 and 0.44 for \( r_c = 25 \) and 50 cm, respectively, correspond to the case for which the sink bottom \((z_{ref})\) touches the zero potential surface, i.e., when \( z_0 = 20 \) cm. The RWUR increases as the distance between \( z_{ref} \) and \( z_0 \) increases, and it decreases with increasing lysimeter width \((r_c)\). For the given soil texture parameter, \( \alpha = 0.04035 \) cm\(^{-1}\), representative of, for example, a fine sand or loamy soil with a capillary potential created by the sink at the reference point, \( t \) represents, for example, a fine sand or loamy soil with a capillary length \((\alpha^{-1})\) of about 25 cm, for lysimeter depths \((z_0)\) as large as 100 cm, the RWURs in the finite-length cylinders of radii \( r_c = 25 \) and 50 cm approach the values of 0.851 and 0.807, respectively, corresponding to infinitely long cylinders of the same radii. For soils with finer textures, this occurs at deeper lysimeter depths and vice versa for coarser textured soils. The effect of the dry lower boundary on the flow field is demonstrated in Fig. 4, which presents pressure head contours and streamlines for the top 50 cm of finite \((z_0 = 50 \) cm, \( \varphi_0 = 0 \); Fig. 4a) and semi-infinite \((\varphi = 0 \) at \( z = \infty \); Fig. 4b) cylinders of radius \( r_c = 25 \) cm. At the upper part of the cylinders, near the interacting source and sink, the iso-\( b \) contours and streamlines for both cases are quite similar, and the slightly larger soil volume inscribed by the dividing streamline (thicker dashed line) in the semi-infinite domain (Fig. 4b) explains the larger RWUR of 85% in that domain compared with the RWUR of just 81% in the finite, DB cylinder (Fig. 4a). The deeper \( b \) contours differ noticeably from each other; in the finite, DB cylinder, the pressure heads decrease downward faster than in the semi-infinite cylinder, and the iso-\( b \) contours also become normal to the cylindrical wall faster. (Below we also evaluate and discuss the effect of \( \varphi_0 \) on the WUR and on the flow fields in the presence of a WT, \( \varphi_0 > 0 \).)

**Water-Uptake Rate of a Subsurface Point Sink from a Water Table**

If the pressure head at the lower boundary of the cylinder, \( h_0 \), is not that of a free WT \((h_0 = 0)\) but is still finite \((h_0 > -\infty, \varphi_0 > 0)\), the WT supplies water to the point sink at a rate that decreases exponentially with decreasing pressure heads at the lower boundary until the WUR practically vanishes at some highly negative \( h_0 \) value (Fig. 5). For any given prescribed pressure head, the WUR is appreciably larger for the wider and shorter cylinders. However, the WUR is not proportional to the water supply area: in the cylinders with a fourfold WT area \((r_c = 50 \) cm), the WUR is higher by just about 50% compared with the narrower \((r_c = 25 \) cm) cylinders.

**Coupled Point Source–Sink Interacting with a Water Table**

Now we explore, a bit more extensively, the complex scenario of interacting point source–point sink–WT characterized by two kinds of water sources—a singular, localized one with a prescribed, constant flow rate \((q_{so})\) and another, planar one with a prescribed constant potential \((\varphi_0 \) or pressure head, \( h_0 \)), both supplying water to the point sink at a sought-after flow rate \((q_{si})\). As stated above, the linearity of the flow equation allows decoupling of the overall source–sink–WT problem (Fig. 2), which also makes the overall WUR of the sink \((q_{si})\) equal to the sum of the two independent contributions by the WT \((q_{si_wt indep})\) and point source (emitter) \((q_{si_so indep})\). In the following, we present computed mass balance results (Fig. 6–9) and depicted flow fields (Fig. 10–12) and discuss the effects of the system parameters \((Z_0, Z_{si}, R_0, R_{so}, K_{si}, b_0, h_0)\) on the overall WUR and on its potential contributions from the surface point source and from the WT.

In the presence of an additional surface point source, the dependence of the overall WUR on the system parameters is different from that in the absence of the point source presented above. Figure 6 presents the dependence of the overall (total) WUR \((q_{si})\) and its two components \((q_{si_wt indep} \text{ and } q_{si_so indep})\) in narrow \((r_c = 25 \) cm) and wide \((r_c = 50 \) cm) cylinders on the prescribed pressure head.
At the WT, its depth held constant at $z_0 = 50$ cm, for root zone parameters held constant at $z_{si} = r_0 = 10$ cm and for a surface point source strength of $1 \text{ L h}^{-1}$. The soil type here and in the rest of the figures (except Fig. 9) is a loam soil with $\alpha = 0.04035 \text{ cm}^{-1}$ and $K_s = 1 \text{ cm h}^{-1}$. The separately evaluated potential contribution of the point source to the total WUR is independent of $h_0$ and larger for the narrower cylinder (Fig. 6). The independently evaluated potential contribution of the WT increases exponentially with increasing $h_0$ and is larger for the wider cylinder, with differences that increase with $h_0$ (Fig. 6). Consequently, the increasing total WUR($h_0$) curves for the two cylinder widths cross each other, being larger for the narrow cylinder at more negative $h_0$ and larger for the wider cylinder under close-to-zero $h_0$ conditions (Fig. 6).

The dependence of the WUR on the size of the conceived root zone, $r_0$ (assuming $z_{si} = r_0$), thus also $z_{ref_so} = 2r_0$), is demonstrated in Fig. 7 for a cylinder of given length ($z_0 = 100$ cm), radius ($r_c = 50$ cm), and prescribed zero pressure head ($h_0 = 0$) at its lower boundary. The contribution of the free WT to the WUR ($q_{si_wt}^{indep}$) increases significantly as the root zone approaches it (increasing $z_{si}$ and $r_0$), whereas the dependence of the point source contribution to the WUR ($q_{si_so}^{indep}$) on $r_0$ (and $z_{si}$) is moderate, exhibiting a shallow local minimum in the simulated scenario. Accordingly, under these circumstances, the overall WUR also increases significantly with increasing root zone size and depth. If instead of locating the reference point for evaluating $q_{si_wt}^{indep}$ at the bottom of the conceived root zone, $z_{ref_wt} = 2r_0$ (here, empty triangles in Fig. 7 and elsewhere), it would have been located at the top of the root zone, $z_{ref_wt} = 0$ (applied only here, solid triangles in Fig. 7), the contribution of the WT to the overall WUR would have been significantly lower, especially for larger root zone radii.
When using finite-depth lysimeters to study processes affected (Duncan et al., 2016; Raij et al., 2018) lysimeters are widely used. Since the shallower WT’s contribution to the WUR increases pressure head conditions similar to those prevailing in open fields to determine crop water consumption and irrigation scheduling. The total WUR of the shallow WT cases increases with \( r_c \) and then stabilizes at larger \( r_c \) values; and it is only slightly higher for the deeper WT. Since the shallower WT’s contribution to the WUR increases more with \( r_c \) than the point source contribution decreases with \( r_c \), the total WUR of the shallow WT cases increases with \( r_c \), whereas for the deeper WT cases, the dependence on \( r_c \) is opposite under the simulated circumstances.

Weighable (Lazarovitch et al., 2006; Marek et al., 2006; Bryla et al., 2010; Tripler et al., 2012; Evett et al., 2015) and drainage (Duncan et al., 2016; Raij et al., 2018) lysimeters are widely used to determine crop water consumption and irrigation scheduling. When using finite-depth lysimeters to study processes affected by the water regimes, care should be taken in interpreting the measurements, especially when inferring the expected behavior under natural, free-drainage field conditions from the lysimeter measurements (Abdou and Flury, 2004; Groh et al., 2016). One approach to coping with the artifactual conditions at the bottom of the lysimeter is to use conductive drainage extensions to create pressure head conditions similar to those prevailing in open fields at the same depths (their features suggested to be determined based on a one-dimensional, vertical flow analysis; Rimmer et al., 1995; Ben-Gal and Shani, 2002). Here we apply the suggested analysis of steady three-dimensional flow, which also accounts for the complex effects of the lateral and vertical confinements, to illustrate how to choose the suction at the bottom of the lysimeter (\( \theta_0 \)) that results in the same WUR as in an otherwise similar, semi-infinite, open-field soil domain. Notice that this evaluation is relevant to the conditions in practical lysimeters (with or without drainage extensions) only when the overall sink strength is smaller than the source strength (\( q_{si} < q_{so} \)) (depicted in Fig. 10 below) because in practical lysimeters the only water source is irrigation (or rain) in the upper region. In this analysis, the \( \theta_0 > -\infty \) lower boundary provides another water source, with an upward flow that compensates for the reduced WUR due to the reflection from the finite \( z_0 \), lower dry boundary (Fig. 3). This simulated situation of net drainage and water circulation from the periphery to the center at the bottom of the lysimeter can appear via a conducting layer positioned at the bottom of the lysimeter. Mind the discrepancy due to the contradictory assumptions of a constant-pressure lower boundary condition in the mathematical analysis and the lateral flow in the lower conducting layer, which is minimal for a (water-saturated) highly conducting drain made of, e.g., rockwool.

**Figure 8.** Water-uptake rate (WUR) for a coupled source–sink in confined cylinders of finite lengths \( z_0 = 50 \) and 100 cm, again for root zone parameters held constant at \( z_{si} = r_0 = 10 \) cm and for a point source of 1000 cm\(^3\) h\(^{-1}\). The independently evaluated contribution of the WT to the WUR is significantly larger for the shallower WT (\( z_0 = 50 \) cm); it first increases steeply with increasing \( r_c \) and then saturates for larger \( r_c \) values. In contrast, the separately evaluated contribution of the surface point source to the WUR first decreases with increasing \( r_c \) and then stabilizes at larger \( r_c \) values; and it is only slightly higher for the deeper WT. Since the shallower WT’s contribution to the WUR increases more with \( r_c \) than the point source contribution decreases with \( r_c \), the total WUR of the shallow WT cases increases with \( r_c \), whereas for the deeper WT cases, the dependence on \( r_c \) is opposite under the simulated circumstances.

**Figure 9.** Prescribed pressure heads at the bottom of finite-length cylinders (Lys) packed with three soil types (loamy sand, capillary length \( a = 0.1276 \) cm\(^{-1}\), saturated hydraulic conductivity \( K_s = 10 \) cm h\(^{-1}\); loam, \( a = 0.04035 \) cm\(^{-1}\), \( K_s = 1 \) cm h\(^{-1}\) cm\(^{-1}\); clay, \( a = 0.0128 \) cm\(^{-1}\), \( K_s = 0.1 \) cm h\(^{-1}\)) that result in the same water-uptake rate as that of a coupled source–sink in equivalent vertically unconfined, semi-infinite cylinders as a function of the depth of the finite cylinders (black solid lines). Also shown are the pressure head distributions along the axis (dashed red lines) and along the wall (dotted blue lines) of the vertically unconfined cylinders packed with the same soil types. The evaluated sink strengths (\( q_{si}, \) cm\(^3\) h\(^{-1}\)) common to both the finite and semi-infinite cylinders of the three soil types are shown near the curves. The radius of the cylinders is \( r_c = 50 \) cm. The strength of the surface point source is \( q_{so} = 1000 \) cm\(^3\) h\(^{-1}\). The conceived sink is located at depth \( z_{si} = 20 \) cm below the source and has a radius of \( r_0 = 20 \) cm.
Figure 9 presents evaluations of the required, prescribed $h_0$ that results in the same WUR as those in semi-infinite, freely draining cylinders as a function of lysimeter depth ($z_0$) for three different soil textures, with root zone parameters held constant at $z_{si} = r_0 = 20$ cm, a constant lysimeter radius of $r_c = 50$ cm, and a point source of 1000 cm$^3$ h$^{-1}$. In the finite-depth lysimeters, the WUR originating from the surface emitter ($q_{si-so}$) is smaller than in the semi-infinite cylinder, and the WUR from the WT ($q_{si-wt}$) balances this difference. In addition to the required, equivalent pressure heads (black solid lines), the vertical distributions of the pressure head along the axis (dashed red lines) and along the walls (dotted blue lines) in the corresponding semi-infinite soil domains are also plotted. Naturally, in the clayey soil (with maximum WUR of 901 cm$^3$ h$^{-1}$), higher suction (more negative $h_0$) is required to reproduce the open-field WUR. For deeper lysimeters, the pressure heads along the lysimeter’s axis and wall are similar, i.e., the flow converges to a one-dimensional pattern, and the above-mentioned one-dimensional analysis is perhaps sufficient. However, for the shorter cylinders, the two vertical $h$ distributions depart and a three-dimensional analysis is necessary to provide the required equivalent $h$ at the bottom of the lysimeter.

Figures 10 to 12 depict sample flow fields of source–sink–WT systems for which the sink strength ($q_s$) is either smaller than (Fig. 10), larger than (Fig. 11) or exactly equal to (Fig. 12) the source strength ($q_s$). In the first example (Fig. 10), the lower boundary at $z_0 = 50$ cm is held constant at $h_0 = −90.3$ cm (the value providing the same $q_s$ as in an equivalent semi-infinite soil domain, Fig. 9), the surface point source strength is $q_{so} = 1000$ cm$^3$ h$^{-1}$, and the sink strength is $q_{si} = 745$ cm$^3$ h$^{-1}$. This sink strength is the sum of the two independently evaluated, potential contributions: 595 cm$^3$ h$^{-1}$ from the surface point source ($q_{si-so}^\text{indep}$ in the So-Si-DB scenario, for which the deep percolation rate is $1000 − 595 = 405$ cm$^3$ h$^{-1}$) and 150 cm$^3$ h$^{-1}$ from the WT ($q_{si-wt}^\text{indep}$ in the Si-WT problem). However, in the combined depicted So-Si-WT scenario, the point source contribution to the overall sink WUR is larger, 737 cm$^3$ h$^{-1}$, whereas the contribution of the WT is significantly smaller, only 8 cm$^3$ h$^{-1}$ (evaluated by integrating the water flux at $z_0$ between $r = 0$ and the stagnation point at $r = 13$ cm). This is why this weak upward flow from the WT to the sink between the cylinder axis and the stagnation point is not visible with the coarse streamline resolution of Fig. 10. The deep percolation from the surface point source to the WT in the peripheral region ($13 < r < 50$ cm) of the depicted flow field of the combined So-Si-WT problem is 263 cm$^3$ h$^{-1}$ (evaluated by integrating the water flux at $z_0$ between $r = 13$ and $r_c = 50$ cm), significantly smaller than in the So-Si-DB problem (405 cm$^3$ h$^{-1}$). The net outflow from the bottom of the lysimeter, 255 cm$^3$ h$^{-1}$ (263 − 8 cm$^3$ h$^{-1}$), equals the difference between the point source strength and the overall sink strength ($1000 − 745$ cm$^3$ h$^{-1}$). The depicted pressure head distribution, $h(r,z)$, is computed from the MFP distribution, $\varphi_{si-so-wt}(r,z)$, which is the sum of three

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**Fig. 10.** Lines of constant pressure head (cm, solid lines) and streamlines (dashed lines) for a coupled source–sink in a finite cylinder of radius $r_c = 50$ cm and length $z_0 = 50$ cm with a prescribed pressure head of $h_0 = −90.3$ cm at its lower boundary (this boundary condition results in the same water-uptake rate as in a semi-infinite cylinder in the loam soil, Fig. 9). The conceived sink of radius $r_0 = 20$ cm is located at depth $z_{si} = 20$ cm. The source strength is $q_{so} = 1000$ cm$^3$ h$^{-1}$ and the evaluated sink strength is $745$ cm$^3$ h$^{-1}$, of which 737 cm$^3$ h$^{-1}$ comes from the point source and 8 cm$^3$ h$^{-1}$ is contributed by the water table. The actual sink is shaded. The thicker dashed line denotes the streamlines that divide the soil domain into three sections: (left to right) an inner soil volume in which water flows upward from the water table toward the sink, an intermediate volume through which water is taken up from the surface point source, and an external volume of deep percolation from the surface point to the water table.

**Fig. 11.** Lines of constant pressure head (cm, solid lines) and streamlines (dashed lines) for a coupled source–sink in a finite cylinder of radius $r_c = 50$ cm and length $z_0 = 100$ cm with a free water table ($h_0 = 0$) at its bottom. The conceived sink of radius $r_0 = 20$ cm is located at depth $z_{si} = 20$ cm. The source strength is $q_{so} = 1000$ cm$^3$ h$^{-1}$ and the evaluated sink strength is $1166$ cm$^3$ h$^{-1}$, of which 1000 cm$^3$ h$^{-1}$ comes from the point source and 166 cm$^3$ h$^{-1}$ is contributed by the water table. The actual sink is shaded.
components: (i) the background, hydrostatic potential due to the prescribed constant pressure head at the WT (\(\varphi_{\text{wt}}(z)\), a vertical, linear distribution of upwardly decreasing pressure head—a constant-value total hydraulic head); (ii) the three-dimensional potential field generated by the surface point source [of strength 1000 cm\(^3\) h\(^{-1}\), \(\varphi_{\text{so}}(r,z)\)]; and (iii) the three-dimensional potential field generated by the subsurface point sink [of strength 745 cm\(^3\) h\(^{-1}\), \(\varphi_{\text{si}}(r,z)\)].

The flow field in another illustrated source–sink–WT system for which the sink strength (\(q_{\text{si}} = 1166 \text{ cm}^3 \text{ h}^{-1}\)) is now larger than that of the surface point source (\(q_{\text{so}} = 1000 \text{ cm}^3 \text{ h}^{-1}\)) is depicted in Fig. 11. Here the depth of the WT with a prescribed pressure head of \(h_0 = 0\) is \(z_0 = 100\) cm. The independently evaluated, potential contribution of the WT to the sink (\(q_{\text{wt}}^{\text{indep}}\)) is 435 cm\(^3\) h\(^{-1}\) and that of the point source (\(q_{\text{so}}^{\text{indep}}\)) is 731 cm\(^3\) h\(^{-1}\). However, inspecting the flow field (streamlines, Fig. 11) reveals that in this scenario of higher overall sink strength than surface point strength, the independently evaluated excess deep percolation rate from the source in the So-Si-DB problem (269 cm\(^3\) h\(^{-1}\)) is fully eliminated by the independently evaluated upward flow rate from the WT (435 cm\(^3\) h\(^{-1}\)). Thus, in this combined So-Si-WT scenario, there is no deep percolation from the surface point source to the WT, i.e., the point source contribution to the overall sink WUR is 1000 cm\(^3\) h\(^{-1}\) and that of the WT is just 166 cm\(^3\) h\(^{-1}\) (in other \(q_{\text{si}} > q_{\text{so}}\) circumstances of, e.g., a deeper lysimeter, drainage from the surface point source to the WT at the periphery of the lysimeter can still occur, which cannot be determined based on only the independently evaluated \(q_{\text{si}}^{\text{wt}}^{\text{indep}}\) and \(q_{\text{so}}^{\text{indep}}\)). Only upward flow toward the point sink is seen in the lower part of the soil domain, whereas in its upper part, the whole flow converges into the sink. A \(q_{\text{si}} > q_{\text{so}}\) scenario can arise with only a lower water source—groundwater under natural conditions. It is also possible to connect the bottom of a lysimeter, protected by a device that prevents air entry, to a constant-head water tank located at a lower (for \(h_0 < 0\) cases such as in Fig. 10 and 12) or the same (for the \(h_0 = 0\) case of Fig. 11) elevation, which supplies the evaluated net upward flow.

Figure 12 depicts the flow field of a special case for which the strength of the subsurface point sink is exactly that of the surface point source (\(q_{\text{si}} = q_{\text{so}} = 1000 \text{ cm}^3 \text{ h}^{-1}\)), which was achieved by iteratively setting the prescribed pressure head to \(h_0 = -65.66\) cm at the \(z_0 = 50\) cm lower boundary. A \(q_{\text{si}}\) of 911 cm\(^3\) h\(^{-1}\) comes from the point source, and 89 cm\(^3\) h\(^{-1}\) (evaluated by integrating the upward water flux from \(r = 0\) to the stagnation point at \(r = 27.5\) cm) is contributed by the WT. This flow field is generally similar to that depicted in Fig. 10 but here, the sink is stronger (1000 cm\(^3\) h\(^{-1}\) compared with 745 cm\(^3\) h\(^{-1}\) in Fig. 10) due to larger contributions of both the surface point source (911 cm\(^3\) h\(^{-1}\) compared with 738 cm\(^3\) h\(^{-1}\) in Fig. 10) and the WT (89 cm\(^3\) h\(^{-1}\) compared with 8 cm\(^3\) h\(^{-1}\) in Fig. 10). This is visible in the steeper pressure head gradients from the WT toward the sink. This simulated flow field represents a unique situation of full circulation at the bottom of the lysimeter with neither net downward drainage nor upward flow into the lysimeter: 89 cm\(^3\) h\(^{-1}\) percolates from the surface point source to the WT with exactly the same upward flow rate from the WT to the point sink.

**Summary and Conclusion**

The previously derived analytical solution for steady infiltration from a point source into a finite cylindrical domain was used to analyze the effects of geometry, soil type, and lower boundary conditions on the flow fields and WUR of subsurface point sinks, with or without a surface point source in the presence of a shallow WT. This analysis is relevant to the water flow and uptake processes occurring in either natural fields with shallow groundwater (Wang et al., 2018) or artificial lysimeters (Groh et al., 2016). The linearized steady water flow equation and the suggested, coupled source–sink–WT approach allow evaluating the two independent—point source and WT—contributions to the WUR when acting separately and also both contributions when acting together. This is useful for common situations in which the groundwater and the drip irrigation water are of different cost and quality (e.g., salinity). When accounting for both simultaneous contributions in the combined source–sink–WT problem, the contribution of the point source to the sink strength is larger than that evaluated independently, whereas that of the WT is smaller than its independently evaluated counterpart. It should be borne in mind that the analysis here evaluated the maximum possible simultaneous water uptake from both the surface point source and the WT, assuming that the latter does not affect the uptake from the surface source and vice versa. This analysis also does not account for these sources’ potentially different quality. It also assumes
maximum possible suction at the root zone and therefore ignores the plant–atmosphere resistance to water uptake and refers only to the resistance (or ability) of the soil to conduct water from a surface emitter or from shallow groundwater to the root zone. In this respect, the evaluated WURs should be regarded as the maximum possible ones. The reference to steady flow and uptake processes also does not correspond directly to the real, unsteady processes. Nevertheless, based on the approach that was outlined and discussed in, for example, Communar and Friedman (2010, 2011) and Friedman et al. (2016) (in the absence of shallow groundwater), the present analysis can be used to evaluate the effects of WT depth ($z_0$), soil properties ($\alpha$ and $K'$), the depth and size of the root zones ($z_{rit}$, $r_0$), and distances between the plants (approximating the area per plant with $\pi r^2$) on the potential utilization of shallow groundwater. Under circumstances of a supplementary drip irrigation system, the presented computations can also support design decisions concerning distances between the emitters or plants and the irrigation rates required to complement water uptake from the groundwater. In principle, the general solution for a steady, subsurface, point source in a confined cylinder (Eq. [6]), derived in Communar and Friedman (2015) can be used also for analyzing the problem of an interacting subsurface emitter–root system–WT system. However, in most circumstances, it is not agronomically sensible to install a subsurface drip irrigation system over shallow groundwater. To some extent, the analysis presented here can also assist in choosing the proper prescribed pressure heads at the bottom of lysimeters ($h_0$) for maximum similarity to conditions in freely draining natural fields (Rimmer et al., 1995; Ben-Gal and Shani, 2002; Groh et al., 2016). In the near future, we plan to implement the computations presented here in the DIDAS program (Communar et al., 2015; Friedman et al., 2016).

Acknowledgments

Contribution no. 603/18 of the Agricultural Research Organization, Israel. We are thankful to Gregory Communar for his helpful comments. This work was supported by funding from the Chief Scientist of the Israeli Ministry of Agriculture and Rural Development, NARO-ARO Project no. 20-03-0033.

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