Electrical Resistivity of a Partially Saturated Porous Medium at Subzero Temperatures

Teddi Herring,* Edwin Cey, and Adam Pidlisecky

Electrical resistivity tomography has been used in many frozen ground applications to delineate frozen and unfrozen areas of the subsurface and monitor changes with time. In these studies, the amount of unfrozen water remaining in the pore space at subzero temperatures is often a parameter of interest. To interpret resistivity data quantitatively in terms of unfrozen water content, it is necessary to establish a relationship between bulk resistivity, subzero temperature, and liquid water saturation. In the literature, a consensus has not been reached on the form of this relationship, and a better understanding of the mechanisms controlling the resistivity of frozen ground is needed. This study used a unique laboratory apparatus to collect electrical resistivity tomography data for a uniform porous medium at temperatures from −20 to 25°C for a range of initial water saturations. Archie’s equation was modified to include the effects of temperature above and below 0°C. Resistivity data collected below 0°C were used to estimate temperature-dependent liquid water saturation and fluid resistivity. The amount of unfrozen water remaining at a given temperature was not related to initial water saturation and was nearly identical for all initial saturations at temperatures below about −5°C. The dependence of resistivity–temperature curves on initial water saturation at subzero temperatures was caused by differences in fluid resistivity as a result of ion exclusion during freezing. The relationships established in this study provide insight into the physical mechanisms that govern the resistivity of porous media at subzero temperatures and a starting point for quantitative analysis of resistivity data collected in frozen ground.

Abbreviations: ERT, electrical resistivity tomography; SFCC, soil freezing characteristic curve.

Electrical resistivity tomography (ERT) is a minimally invasive and spatially exhaustive geophysical method that produces an image of subsurface resistivity. The resistivity of earth materials depends on several factors including temperature, water saturation, and resistivity of the pore fluid. Electrical resistivity tomography is well suited to situations where the target of a geophysical survey has a different resistivity than the surrounding material. The applications of ERT are diverse and include imaging temperature contrasts for geothermal (Hermans et al., 2012) or hydrological applications (Musgrave and Binley, 2011), mapping variation in saturation to make inferences about surface water infiltration (Daily et al., 1992) or water uptake by crops (Michot et al., 2003), and imaging conductive solutes for tracer tests (Singha and Gorelick, 2005), contaminant imaging (Chambers et al., 2006), and remediation (Oldenborger et al., 2007). Recent concerns about the impact of thawing permafrost on atmospheric CO2 and CH4 levels (Schuur et al., 2008), alpine slope stability (Noetzli et al., 2003), and infrastructure integrity (Greenslade and Nixon, 2000) have spurred the use of ERT in many frozen ground monitoring studies to map the distribution of permafrost (Lewkowicz et al., 2011; McClymont et al., 2013; Kneisel et al., 2014a) and image the evolution of frozen and unfrozen areas (Krautblatter and Hauck, 2007; Kneisel et al., 2014b; Oldenborger and LeBlanc, 2018). Interpreting resistivity data in terms of unfrozen water content has been a goal in many permafrost studies (Hilbich et al., 2008; Dafflon et al., 2016; Keating et al., 2018; Oldenborger and LeBlanc, 2018). Quantitative inferences about unfrozen water content require rock physics...
relationships among electrical resistivity, temperature, and liquid water saturation.

The temperature dependence of soil resistivity in the 0 to 25°C range is well established and has been used in many studies to mitigate the effect of temperature so that bulk resistivity can be interpreted in terms of fluid resistivity or saturation (Hayley et al., 2007; Jayawickreme et al., 2008; Hayley et al., 2009, 2010). The temperature correction in these studies relies on the predictable relationship between resistivity and temperature between 0 and 25°C. However, the bulk temperature–resistivity relationship under frozen conditions is more complex and has not been thoroughly studied. Previous laboratory measurements of soil or rock resistivity at subzero temperatures yielded relationships of different forms that vary significantly depending on the type of material (Harlan et al., 1971; McGinnis et al., 1973; King, 1977; Hauck, 2001; Krautblatter et al., 2010). Quantifying a reliable temperature–resistivity relationship is necessary to remove the effects of temperature to estimate other parameters.

Under frozen conditions, the relationship between temperature and bulk resistivity is controlled by unfrozen water in the pore space, but the physical processes that govern the amount and distribution of liquid water at subzero temperatures are not well understood. The relationship between liquid water content and temperature, commonly referred to as the soil freezing characteristic curve (SFCC), has been described with empirical functions of various forms including power law (Anderson and Tice, 1972), exponential (Michalowski and Zhu, 2006), piecewise linear (McKenzie et al., 2007), and nonlinear piecewise functions (Kozlowski, 2007). The SFCCs vary depending on the soil type, with differences often related to the soil’s specific surface area (Dillon and Anderliland, 1966; Anderson and Tice, 1972; Kozlowski, 2007) or the pore size distribution (Liu and Yu, 2013; Wang et al., 2017).

Additionally, there has been some debate as to whether the initial water saturation affects unfrozen water content at temperatures below zero. Some studies have observed higher liquid water saturation with higher initial saturations (Suzuki, 2004; Wen et al., 2012; Shan et al., 2015), while others saw no trend between unfrozen water content and initial saturation (Watanabe and Wake, 2009; Zhou et al., 2014; Tang et al., 2018). Some have argued that failing to account for the ice phase in nuclear magnetic resonance and time-domain reflectometry studies leads to systematic overestimates of liquid water saturation for higher initial saturations, and that when appropriate corrections are applied, liquid water saturation does not depend on initial saturation (Watanabe and Wake, 2009; Zhou et al., 2014). The existing literature is not clear consensus on the form of the SFCC or its physical basis, and consequently the relationship between temperature and resistivity at subzero temperatures remains an unsolved problem.

In this study, we used a simple medium of unconsolidated glass beads and collected ERT data at temperatures between −20 and 25°C at five different initial saturations. Our objectives were twofold: (i) to methodically establish the relationships among bulk resistivity, temperature, and initial water saturation as well as meaningful estimates of uncertainty, and (ii) to estimate fluid resistivity and unfrozen water content from resistivity data using a modified form of Archie’s equation. The results of this analysis provide a basis for a generalized temperature–resistivity relationship at subzero temperatures and yield useful insight into the physical mechanisms behind the observed bulk resistivity data.

Theory

Empirical Resistivity Relationships

Archie’s equation (Archie, 1942) is an empirical relationship that describes the bulk resistivity $\rho_{\text{bulk}}$ (Ω m) of clay-free materials as a function of porosity $\phi$ (m3 m−3), water saturation $S_w$ (m3 m−3), and fluid resistivity $\rho_f$ (Ω m) using unitless fitting parameters $m$ (the cementation exponent) and $n$ (the saturation exponent). A slightly modified version of Archie’s equation, which is more commonly used in practice (Winsauer et al., 1995), includes another empirical fitting parameter $a$, often called the tortuosity factor:

$$\rho_{\text{bulk}} = a\phi^{-m}S_w^{-n}\rho_f$$  \[1\]

It has been argued that the inclusion of a non-unity value of $a$ in Archie’s equation is theoretically unjustified because it causes a mathematical inequality when $\phi \to 1$. However, Glover (2016) demonstrated that a non-unity value of $a$ results in a better data fit in practice by compensating for systematic errors in measured porosity, pore fluid salinity, and temperature. It can also be argued that, being an empirical relationship, Archie’s equation is expected to be valid only across the porosity range represented by the data and that the apparent paradox introduced by a non-unity $a$ when $\phi \to 1$ is irrelevant (Worthington, 1993).

Archie’s equation does not explicitly include the effect of temperature on resistivity measurements. The effect of temperature on the bulk resistivity in the 0 to 25°C range is well constrained and is directly related to the fluid resistivity, which decreases at higher temperature due to decreased viscosity (Wagner, 2012). A temperature-dependent fluid resistivity can easily be incorporated into Archie’s equation. The theoretical temperature dependence of resistivity for electrolytic solutions has been described in the literature (Keller and Frischknecht, 1966) as

$$\rho_f = \rho_{f,\text{ref}}\left[d(T-T_{\text{ref}}) + 1\right]^{-1}$$  \[2\]

where $\rho_f$ (Ω m) is the measured fluid resistivity at temperature $T$ (°C), $\rho_{f,\text{ref}}$ (Ω m) is the fluid resistivity at a reference temperature $T_{\text{ref}}$ (°C) (typically 25°C), and $d$ (°C−1) is a constant temperature compensation factor. Estimates of $d$ are typically close to 0.02 for most ionic solutions, with some variability, e.g., $d = 0.0183$°C−1 (Hayley et al., 2007), $d = 0.0187$°C−1 (Hayashi, 2004), $d = 0.0191$°C−1 (Clesceri et al., 1998), $d = 0.0202$°C−1 (Campbell et al., 1948), $d = 0.025$°C−1 (Keller and Frischknecht, 1966).

At subzero temperatures, the bulk resistivity–temperature relationship is not well defined. Some studies (McGinnis et al.,
1965; Hauck, 2001) have fit experimental bulk resistivity data in the −20 to 0°C range with exponential curves. Others have observed non-exponential relationships (Harlan et al., 1971), particularly at temperatures close to 0°C (Krautblatter et al., 2010).

Physical Model of Freezing

The relationship between resistivity and temperature under frozen conditions depends on the amount of water that remains unfrozen in the pore space. The unfrozen water content also controls fluid resistivity as ions are excluded from the lattice structure of ice during freezing and concentrated in the remaining liquid. The persistence of unfrozen water at temperatures below 0°C was hypothesized to be a result of sorptive and capillary forces causing a depression in the freezing point (Miller, 1965), a theory that has been verified experimentally (Koopmans and Miller, 1966; Cahn et al., 1992). In clays, the unfrozen water depends mostly on sorptive forces, while capillary forces are dominant in granular materials (Koopmans and Miller, 1966). In granular material, the freezing point depression is inversely related to the pore radius, i.e., fluid in larger pores freezes first (Stähli et al., 1996), with a thin film of water remaining around grains (Taber, 1929).

The similarities between the SFCC and soil moisture characteristic (the relationship between soil moisture and pore water suction) have been presented in many studies (Koopmans and Miller, 1966; Black and Tice, 1989; Spaans and Baker, 1996; Florschue et al., 2017), and the distribution of fluid in frozen soils and dry soils is understood to be analogous. Using this paradigm, the increase in resistivity during freezing can be modeled as a decrease in liquid water saturation (Daniels et al., 1976; Hauck, 2002; Minsley et al., 2015; Dafflon et al., 2017).

Unfrozen Water Content and Fluid Resistivity

Framing the freezing process as a reduction in water saturation means that Archie’s equation can be applied at subzero temperatures with two modifications. First, a new variable for liquid water saturation is defined for temperatures below 0°C:

\[ S_{wL}(T) = S_s(T)S_{w0} \]  

where liquid water saturation \( S_{wL}(T) \) (m\(^3\) m\(^{-3}\)) depends on temperature via the SFCC. Relative saturation \( S_s(T) \) is the dimensionless ratio of unfrozen water saturation to initial water saturation \( S_{w0} \) (m\(^3\) m\(^{-3}\)). Archie’s equation for freezing materials can be altered by replacing \( S_w \) with \( S_{wL}(T) \) in Eq. [1].

As pore fluid freezes and ice forms, solutes are concentrated in the remaining liquid water due to ion exclusion, which alters the fluid resistivity. The second modification to Archie’s equation therefore involves defining the dependence of fluid resistivity on the relative saturation. To specify this relationship, it is necessary to quantify how solute (in this case, NaCl) concentration affects electrical resistivity. The relationship between \( \rho_{f,25} \) (Ω m) and NaCl concentration \( C \) (g L\(^{-1}\)) at 25°C can be quantified by fitting tabulated values (Invensys Foxboro, 1999):

\[ \rho_{f,25} = \left(0.159C_0 \right)^{-1} \quad [4] \]

where \( C \) is related to the initial NaCl concentration \( C_0 \) (g L\(^{-1}\)) and the relative saturation \( S_s \) as

\[ C = \frac{C_0}{S_s} \quad [5] \]

Equation [5] assumes that all solute is excluded from the solid ice and is concentrated in the remaining liquid water. Some field-scale geophysical studies have added an exponent to \( S_s \) that accounts for the loss of solute from the liquid phase via diffusion or other transport processes (Minsley et al., 2015; Dafflon et al., 2017). However, in the closed system under consideration in this experiment, full exclusion of ions from the ice phase was expected to lead to an equal increase of ions in the fluid phase, as evidenced by studies of porous media (Banin and Anderson, 1974) and sea ice (Eicken, 2008; Hunke et al., 2011). Combining Eq. [4] and [5] yields the following relationship between fluid resistivity and relative saturation at a standard temperature of 25°C:

\[ \rho_{f,25} = \frac{S_s}{0.159C_0} \quad [6] \]

Other studies have scaled fluid resistivity to include the viscosity-related temperature dependence below 0°C using Eq. [2] (Minsley et al., 2015; Wu et al., 2017). However, the validity of extrapolating Eq. [2] to subzero temperatures is questionable. If \( d = 0.02°C^{-1} \), Eq. [2] yields physically impossible negative fluid resistivities below −25°C. We expect that a more realistic fluid resistivity–temperature relationship would approach an asymptote at some subzero temperature. In the absence of data to describe this relationship, we chose to use Eq. [2] to calculate the fluid resistivity at \( T = 0°C \) without incorporating further temperature dependence under frozen conditions:

\[ \rho_f = \begin{cases} 
\frac{1}{0.159C_0(d(T-25)+1)} & \text{if } T \geq 0°C \\
\frac{S_s}{0.159C_0(d(0-25)+1)} & \text{if } T < 0°C 
\end{cases} \quad [7] \]

If the fluid resistivity at subzero temperatures is underestimated using Eq. [7] then the subsequent estimate of relative saturation will be overestimated. This method therefore provides an upper bound on liquid water saturation, with a higher degree of uncertainty at lower temperatures.

Conceptualizing freezing as a reduction in saturation means that Archie’s equation can be modified so that the bulk resistivity at subzero temperatures depends on the liquid water saturation and the resistivity of the liquid water. Substituting Eq. [3] and [7] into Eq. [1], the final expression for bulk resistivity then becomes

\[ \rho_{\text{bulk}} = \begin{cases} 
\rho_f^{-m} S_{w0}^{-m} \frac{1}{0.159C_0(d(T-25)+1)} & \text{if } T \geq 0°C \\
\rho_f^{-m}(S_sS_{w0})^{-m} \frac{S_s}{0.159C_0(d(0-25)+1)} & \text{if } T < 0°C 
\end{cases} \quad [8] \]
Assuming known values of porosity and Archie fitting parameters $a$, $m$, and $n$, initial water saturation $S_{w0}$, and NaCl concentration $C_0$, we can solve for $S_r$ at any subzero bulk resistivity:

$$S_r = \left[ \frac{\rho_{\text{bulk}, T < 0}}{0.159C_0 \left[ d(0-25) + 1 \right]} a^{-1} \phi^{-m} S_{w0}^{-n} \right]^{1/(1-n)} \quad [9]$$

Any value of bulk resistivity can now be used to calculate a unique $S_r$, and the resulting liquid water saturation and fluid resistivity can be estimated.

Methods

The experimental workflow is shown in Fig. 1 and is described in detail below.

Data Acquisition

A shallow box was built with internal dimensions of 28-cm length, 28-cm width, and 3.5-cm depth, yielding a total volume of 2.7 L. Twenty-four ERT electrodes (steel screws, diameter = 0.4 cm) and four type-T thermocouples were installed around the perimeter of the box, with a fifth thermocouple inserted into the center of the apparatus through the lid (Fig. 2). The box was filled with spherical glass grains (Spheriglass 2024A, grain diameter = 106 to 212 μm, porosity listed as 0.37 to 0.42 and measured gravimetrically to be 0.40) that had been rinsed with deionized water until the measured electrical conductivity of the rinsate was equal to that of the input deionized water. Pore water solution was prepared with 0.257 g L$^{-1}$ NaCl and deionized water to achieve an electrical resistivity of 19.8 Ω m (0.0506 S m$^{-1}$), a value that would be expected of relatively fresh groundwater (Sanders, 1998).

The ERT data were collected for bead-fluid mixtures in five trials that had different initial water saturations $S_{w0}$ (m$^3$ m$^{-3}$) of 0.20, 0.40, 0.60, 0.80, and 1.0, across a range of temperatures between approximately −20 and 25°C. In the first trial, 0.22 L of solution was added to the glass beads and thoroughly mixed to yield a homogeneous medium with a saturation of 0.20. The mixture was evenly distributed in the box, which was then sealed to avoid evaporation and sublimation, placed in a freezer, and cooled to approximately −20°C. Data were not collected during cooling. The freezer was then turned off and data were collected during warming. To minimize the rate of temperature change, the freezer was insulated with polystyrene panels, and water jugs were used as thermal mass. Electrical resistivity tomography data were collected approximately every 2°C at temperatures below zero and every 5°C at temperatures above zero using a Wenner configuration with 168 unique data points and three stacked measurements per data point. Each ERT dataset took 6 min to collect using an Iris Instruments Syscal Pro on a constant injection current setting. Temperature measurements were collected at regular 1-min intervals with a Campbell Scientific CR1000 datalogger. Temperature data were averaged temporally across the ERT data collection window and averaged spatially between outer and inner temperatures to yield a single averaged temperature value for each ERT dataset. The first data collection cycle ended when the apparatus reached room temperature. In the second trial, an additional 0.22 L of prepared fluid was then mixed into the medium to achieve a saturation of 0.40, the apparatus was frozen, and data were collected again during warming. This process was repeated in subsequent trials with initial saturations of 0.60, 0.80, and 1.0, resulting in a total of 79 ERT datasets collected across a range of temperatures and initial saturations.

Data Filtering

Each ERT measurement consisted of an injected current value, measured voltage, and standard deviation. Measurements had low coefficients of variation, with a maximum for any one measurement of 1.3%. No measurements were removed based on the coefficient of variation. Data points with voltages exceeding the maximum measurable value were removed. Data filtering by measured voltage resulted in ERT datasets ranging from 77 to 168 individual measurements, with an average of 120 measurements.
Homogeneous Resistivity Analysis

Raw ERT data are measurements of electrical resistance, the ratio of measured voltage to input current, and are dependent on measurement geometry. Resistance measurements must be processed to obtain estimates of the medium’s intrinsic resistivity. An analytical solution was generated using a large number of image sources to model the apparatus boundaries and was used to simulate data for a medium of homogeneous resistivity given electrode locations and box geometry. The misfit between observed data and simulated data indicated how well the homogeneous resistivity approximated the physical medium. For each set of ERT measurements, where the number of measurements is \( n \), the best-fit homogeneous resistivity minimized the average misfit \( E(\%) \) between observed resistance data \( d_{\text{obs}}(\Omega) \) and simulated resistance data \( d_{\text{sim}}(\Omega) \):

\[
E = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{d_{\text{obs}} - d_{\text{sim}}}{d_{\text{obs}}} \right| \times 100\%
\]

For each ERT data set, a range of possible resistivities was defined and a large number of resistivity values were uniformly sampled within this domain. Simulated data were generated for each resistivity model using the analytical solution and compared with the observed data to yield an average absolute data misfit using Eq. [10]. This method yielded the optimal homogeneous resistivity that minimized the data misfit. Determining the range of resistivity values that fit the data to within certain error limits allowed useful estimates of uncertainty. Figure 3 shows an example of how the average misfit between observed and simulated data changes as a function of the input resistivity for one ERT dataset.

Results and Discussion

Temperature

The average of all five temperature sensors was used in the following analysis, but there was some variability among the observed temperature measurements (Fig. 4a), which probably contributed to the heterogeneity in resistivity (discussed below). A relatively large spatial temperature gradient was observed between the edges and center of the apparatus in the 0 to 4°C range (Fig. 4), indicating that equilibrium had not been reached and that the ice in the center of the apparatus had not melted completely. Under these conditions, plateaus in temperature time series also indicated a phase change from ice to water as energy was absorbed into the system with no temperature change (Anderson et al., 1973). This temperature range is indicated as a gray shaded region in the following plots.

Bulk Resistivity–Temperature Curves

Optimal homogeneous resistivities and uncertainties are plotted in Fig. 5 for all initial saturations and temperatures. Bulk resistivity clearly follows different trends under frozen and unfrozen conditions. The discontinuity at 0°C indicates a large increase in bulk resistivity due to freezing of a portion of the pore fluid.
At subzero temperatures. At temperatures >0°C, the decrease in resistivity with warmer temperatures is described by Eq. [2] and is caused by a decrease in fluid viscosity. The average temperature compensation factor $d$ for these data is $0.0215\,°C^{-1}$, which is within the range of values determined in previous studies.

The misfits associated with the optimal homogeneous resistivity models appear to follow the temperature trends seen in Fig. 4b, indicating that some of the heterogeneity in resistivity is attributable to variable temperature within the apparatus. High misfits in the coldest part of the $S_{w0} = 0.2$ curves may also be influenced by high contact resistances.

At temperatures below 0°C, bulk resistivity depends on unfrozen water content and fluid salinity. Using the modified version of Archie’s equation that accounts for the effects of freezing on bulk resistivity, it is possible to estimate liquid water saturation and fluid resistivity separately and examine their relative effect on the observed temperature–bulk resistivity curves below 0°C.

**Archie Fitting Parameters**

To make use of Archie’s equation in this analysis, it was necessary to estimate unitless fitting parameters $a$, $m$, and $n$. It was impossible to determine unique, accurate values for these parameters in this experiment, partially due to a lack of constraining information and partially due to the well-documented uncertainties associated with fitting power-law relationships to sparse datasets (White et al., 2008; Clauset et al., 2009). Instead, we established a range of plausible values for $a$, $m$, and $n$ using a Monte Carlo approach. A large number ($10^6$) of possible combinations of $a$, $m$, and $n$ were randomly selected from given ranges of values with a uniform probability distribution. These values were input into Archie’s equation and used to predict bulk resistivity using the measured porosity ($\phi = 0.40$) and fluid resistivity ($\rho_f = 19.8\,\Omega\,m$). The goal of this exercise was to find an appropriate combination of Archie parameters that could reproduce the observed bulk resistivity values at each initial saturation at a standard temperature of 25°C. Because bulk resistivities were not measured at exactly 25°C, the $T > 0°C$ resistivity curves were fit with a least-squares regression using Eq. [2] and projected to 25°C. The calculated bulk resistivity values were then compared with the observed bulk resistivities, and misfit was calculated using a chi-square metric.

Possible values of $m$ are well constrained in the literature for clean quartz sands and glass beads (Friedman, 2005, and references therein) and range between 1.20 and 1.45. These values of $m$ were used as the input range for the Monte Carlo simulations. Because $a$ and $n$ were not well constrained, they were given very large possible ranges of values ($0.10 < a < 1.5$ and $1.0 < n < 3.0$) guided by literature values (Archie, 1942; Keller and Frischknecht, 1966; Worthington, 1993).

Of the $10^6$ calculated results, we chose to consider the best 100 models rather than the single best model due to the interdependence and non-uniqueness of $a$ and $m$ in this experiment. Given the well-constrained input of $1.20 < m < 1.45$, the values of $a$ and $n$ in the best 100 models were found to be $0.314 < a < 0.405$ and $2.27 < n < 2.31$. The 100 best models produced nearly identical bulk resistivity estimates (Fig. 6). Distributions of bulk resistivities at each of the five saturations calculated using the 100 combinations of $a$, $m$, and $n$ are shown in Fig. 7. The optimal values were taken to be the average of the best 100 models: $a = 0.363$, $m = 1.32$, and $n = 2.29$.

As noted by Glover (2016), a non-unity value of $a$ may be a result of errors in the measurements of porosity, pore fluid salinity (or resistivity), and temperature and can be viewed as an indicator of data quality. The relatively low value of $a$ estimated in this...
study could reflect uncertainties in these parameters. In the Monte Carlo analysis performed in this study, the best-fit value of $a$ was directly related to the input estimate of $m$ because they were interdependent, making it difficult to interpret $a$ solely as a data quality parameter. While the Archie parameter values yield a good fit to our data, their physical significance should not be over-interpreted in this experiment due to the limited amount of constraining data.

**Unfrozen Water and Fluid Resistivity**

Using the modified version of Archie’s equation where liquid water saturation $S_{wL}$ and pore fluid resistivity $\rho_f$ are written explicitly as a function of relative saturation $S_r$, it is possible to calculate $S_r$ for any value of bulk resistivity if the other Archie parameters $\phi$, $a$, $m$, and $n$, initial water saturation $S_{w0}$, and NaCl concentration $C_{0}$ are known (Eq. [9]). The value of $S_r$ can then be translated into unfrozen water saturation via Eq. [3] if the initial water saturation $S_{w0}$ is known. Fluid resistivity $\rho_f$ can also be calculated from $S_r$ if the relationship between solute concentration and fluid resistivity is known and the initial solute concentration or fluid resistivity is known (Eq. [6]). Our results are shown in Fig. 8.

**Sensitivity to Archie Parameters**

The Monte Carlo analysis described above computed 100 combinations of $a$, $m$, and $n$ that fit the data to approximately the same threshold but yielded a significant range of possible values of these parameters. The best estimate of liquid water saturation and fluid resistivity used the average of these 100 values. To assess the sensitivity of the calculated liquid water saturation and fluid resistivity to changes in the Archie parameters, we performed the same calculations using the minimum and maximum values of $a$, $m$, and $n$, yielding the upper and lower bounds on liquid water saturation and fluid resistivity. The results are represented as error bars on the estimates shown in Fig. 8. Using the minimum and maximum values for these parameters shifts the calculated liquid
water saturation and fluid resistivity curves higher or lower, but the qualitative interpretation remains unchanged.

The estimates of liquid water saturation and fluid resistivity as a function of temperature (Fig. 8) contribute valuable insights into the mechanisms that control bulk resistivity at subzero temperatures. The increase in bulk resistivity with decreasing temperatures below zero is due to the decrease in liquid water saturation. The liquid water saturation is independent of initial saturation in Fig. 8a, regardless of the Archie parameter values used. However, the resistivity of the unfrozen fluid does depend on the initial saturation, as a higher initial saturation means a larger solute mass is present, which translates to a higher concentration of solutes in the unfrozen water and therefore a lower fluid resistivity (Fig. 8b). To summarize, the shape of the bulk resistivity–temperature curves below 0°C in Fig. 5 is due to decreasing water content at lower temperatures, which is independent of initial saturation. Variability in these curves at different initial saturations is a result of the salinity of the unfrozen fluid caused by ion exclusion during freezing.

The liquid water saturation curves in Fig. 8a have interesting implications for the current debate on whether the SFCC is affected by the initial saturation. Moisture content in frozen soils is typically inferred using time-domain reflectometry (Spaans and Baker, 1996; Nagare et al., 2012) or nuclear magnetic resonance (Watanabe and Mizoguchi, 2002; Tian et al., 2014). Some studies have demonstrated that failing to account for the ice phase signal may have caused interpretation errors in previous works and that improving the dielectric mixing model or incorporating other data sources like gamma-ray attenuation leads to SFCCs that are independent of initial saturation (Watanabe and Wake, 2009; Zhou et al., 2014). There is also uncertainty in soil moisture inherent in each measurement method and its calibration procedures (Yoshikawa and Overduin, 2005). Inferring soil moisture from resistivity measurements is a novel way of approaching the problem that circumvents these issues. Results...
from this analysis indicate that when estimated from bulk resistance data, unfrozen water content is indeed independent of initial saturation.

Uncertainty

There are several possible sources of uncertainty in this experiment. The uncertainty associated with the raw measurements is low. Resistance data had low coefficients of variation, and measurement stacking was used to reduce noise. The thermocouple manufacturers specify the standard accuracy of the type-T thermocouples used in this experiment as ±1.0°C, but as the absolute error in thermocouple measurements typically manifests as a shift above or below the true temperature, relative changes in temperature are expected to have smaller uncertainty. Averaging temperature data spatially and temporally also reduces error. It is also worth noting that the spatial errors associated with electrode and temperature sensor locations are negligible due to the apparatus being machine built.

Another source of uncertainty is the fact that thermal equilibrium was not reached during data collection. While temporal temperature variation during each ERT data collection window was minimal (at most 0.3°C, typically much less), a spatial temperature gradient existed between the outer edge of the apparatus and the center, as shown in Fig. 4. This probably contributed to the misfits between the observed data and the data simulated using a homogeneous model.

Lastly, previous observations of the relationship between bulk resistivity and temperature <0°C have shown a dependence on the freezing history (Krautblatter et al., 2010; Kang and Lee, 2015). It is expected that the bulk resistivity–temperature curves at subzero temperatures will vary depending on whether the medium is freezing or thawing; however, an in-depth examination of freeze–thaw resistivity hysteresis is beyond the scope of this study.

Conclusions

This study quantified the bulk resistivity–temperature relationship for unconsolidated spherical glass beads at different initial saturations for a range of temperatures above and below 0°C. The two domains of the bulk resistivity–temperature relationship above and below 0°C are governed by different physical phenomena. Above 0°C, bulk resistivity is controlled by fluid resistivity, which decreases predictably with increasing temperature due to decreased fluid viscosity. Below 0°C, the bulk resistivity depends on the amount of unfrozen liquid water and the resistivity of the unfrozen fluid. There has been some debate in the literature over the influence of initial saturation on the unfrozen liquid water at subzero temperatures. When estimated here from bulk resistivity data, the amount of unfrozen water is independent of initial saturation. Variability in the temperature–bulk resistivity curves with initial saturation at subzero temperatures is due to different amounts of solute being present in the unfrozen water.

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