Modeling the Water and Energy Balance of Vegetated Areas with Snow Accumulation

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The ability to quantify soil–atmosphere water and energy exchange is important in understanding agricultural and natural ecosystems, as well as the earth’s climate. We developed a one-dimensional vertical model that calculates solar radiation, canopy energy balance, surface energy balance, snowpack dynamics, soil water flow, and snow–soil–bedrock heat exchange, including soil water freezing. The processes are loosely coupled (solved sequentially) to limit the computational burden. The model was applied to describe water and energy dynamics for a northeast-facing mountain slope in the Dry Creek Experimental Watershed near Boise, ID. Calibration was achieved by optimizing the saturated soil hydraulic conductivity. Validation results showed that the model can successfully calculate seasonal dynamics in snow height, soil water content, and soil temperature. Both the calibration and validation years confirmed earlier results that evapotranspiration on the northeast-facing slope consumes approximately 60% of yearly precipitation, while deep percolation from the soil profile constitutes about 40% of yearly precipitation.

Modeling of water and energy fluxes in snow-dominated mountainous terrain is particularly challenging. The presence of snow modifies the land surface energy balance considerably. Fresh new snow in particular has a high albedo and a low thermal conductivity, which limits daytime soil warming and nighttime soil cooling. Snow is a complicated medium due to continuously changing properties such as grain size, density, and height. Snow modeling concepts vary from relatively simple single-layer representations (e.g., UEB, Tarboton and Luce, 1996; COUP, Jansson and Karlberg, 2004), to more advanced two-layer representations (e.g., Marks et al., 1998; Koivusalo et al., 2001), to sophisticated multilayer numerical approaches (Anderson, 1976; SNThERM, Jordan, 1991; Lehnig et al., 2006).

Soil freeze–thaw may have an important impact on the water and energy fluxes in mountainous terrain. This is especially true during periods in which the snow cover is limited so that the soil is exposed to the atmosphere. Freezing of soil water produces heat, keeping the soil close to 0°C. In contrast, the melting of soil ice requires energy, which delays soil warm-up during spring. Most current soil freeze–thaw algorithms are based on the Clausius–Clapeyron equation, which is used to relate the freezing point of soil water to soil water potential (Fuchs et al., 1978; Spaans and Baker, 1996; Koren et al., 1999; Niu and Yang, 2006).

Snow can be included in vadose zone models using simple degree day concepts (e.g., HYDRUS, Simunek et al., 2005). More physically based methods for modeling snow accumulation involve calculating the surface energy balance. The most sophisticated approaches calculate both the canopy energy balance and the ground surface energy balance (e.g., SHAW, Flerchinger, 1998).
The model solution strategy, whereby governing equations are solved both for leaf temperature and for ground surface temperature. The SHAW model uses relatively simple all-wave expressions to calculate direct and diffuse incoming solar radiation. This all-wave or broadband approach ignores the fact that the albedos of leaves, snow, and soil are all wavelength dependent (Wiscombe and Warren, 1980; Sellers, 1985; Bonan, 1996). The effect of wavelength on the canopy and surface energy balances is included in CLM. This model, however, requires coupling to a global circulation model to obtain accurate estimates of the incoming solar radiation.

We have developed a new model for studying the water and energy balance of mountainous areas that are subject to snow accumulation and melt. A detailed parameterization of the energy fluxes in the soil–plant–atmosphere system was adopted to maximize the model’s ability to accurately describe the timing of snowmelt. This was achieved by combining the detailed solar spectrum model of Bird and Riordan (1986) with comprehensive canopy and surface energy balance calculations taken primarily from CLM. To our knowledge, the Bird–Riordan model has not been used before for vadose zone modeling. The single-cloud model of Munro and Young (1982) was used to describe the effect of clouds on the solar radiation. The effect of complex terrain (slope and aspect) on the incoming solar radiation is also incorporated.

Snow in the model is described using a multilayer approach to account for the often nonlinear temperature distribution in this medium. The treatment of bedrock, soil, and snow as a continuum in the vertical heat transport calculation is novel compared with existing vadose zone and land surface models, which generally do not specifically account for the presence of bedrock. The incorporation of bedrock is important in mountainous areas because of the generally shallow soils combined with the moderating effect of bedrock heat storage on soil temperature fluctuations. The model solution strategy, whereby governing equations are loosely coupled rather than tightly coupled, is similar to the solution strategy used in CLM.

The specific objectives of this study were: (i) to develop a computer simulation model that describes the vertical water and energy fluxes between the soil and the atmosphere in snow-dominated, vegetated areas in a detailed yet computationally efficient way; and (ii) to apply the model to a mountain slope to study the effect of snow accumulation on the annual water and energy balance. The motivation for this study was to develop and test an algorithm that could be applied in a spatially distributed way to quantify runoff generation in small, snow-dominated, mountainous catchments. The distributed model application will be a future topic.

Theory

A vertical one-dimensional model was developed to describe the water and energy balance of vegetated areas subject to snow accumulation and melt. Incoming shortwave radiation is estimated using the solar spectral model of Bird and Riordan (1986). Separate energy balance calculations are conducted for the canopy and the ground surface, following the approach used in CLM (Oleson et al., 2004). Snow water flow and storage is calculated assuming gravity flow only. Vertical soil water flow and storage is based on a noniterative solution of Richards’ equation following Ross (2003). Vertical heat flux and storage in the snow–soil–bedrock is based on the general heat transport equation. Snow and soil water phase change (between liquid water and ice) is determined separately from the water flow and heat transport calculations. Time stepping in the model is on the order of 15 min except for the soil water flow calculation, which may use smaller time steps. The governing equations are described below.

Precipitation

Meteorological input data include precipitation, relative humidity, air temperature, wind speed, and (calculated) cloud cover. Precipitation $p$ is partitioned into rain and snow using air temperature $T_a$:

$$ p_{sn} = p \quad T_a \leq T_{min} \quad \text{[1a]} $$

$$ p_{sn} = \frac{T_{max} - T_a}{T_{max} - T_{min}} p \quad T_{min} < T_a < T_{max} \quad \text{[1b]} $$

$$ p_{sn} = 0 \quad T_a \geq T_{max} \quad \text{[1c]} $$

with

$$ p_t = p - p_{sn} \quad \text{[1d]} $$

where $p_t$ and $p_{sn}$ are the rain and snow rates, respectively. Typical values for the minimum and maximum threshold air temperatures are $T_{min} = -1^\circ C$ and $T_{max} = 3^\circ C$ (U.S. Army Corps of Engineers, 1956). A list of symbols used here is given in Appendix C.

Incoming Shortwave and Longwave Radiation

Incoming shortwave (solar) radiation is calculated in four steps. First, clear-sky (no clouds) direct and diffuse solar radiation is determined using the spectral algorithm of Bird and Riordan (1986). Second, a single-layer cloud model is used to incorporate the effect of clouds (e.g., Munro and Young, 1982). Third, Hay’s model is used to calculate slope irradiance (Muneer, 1997). Finally, the spectral estimates of direct and diffuse solar radiation are summed for the visible ($<0.7 \mu m$) and near-infrared ($0.7-0.7 \mu m$) wavebands. The distinction between direct and diffuse light is important for assessing the effect of terrain slope and aspect on the energy balance. Visible and near-infrared solar radiation is treated separately because of the associated differences in surface albedo.

Clear-sky direct irradiance on a ground surface normal to the direction of the sun $I_{dir0n}$ for wavelength $\lambda$ is given by (Bird and Riordan, 1986)

$$ I_{dir0n}(\lambda) = I_0(\lambda) d_{es} o(\lambda) w(\lambda) m(\lambda) R(\lambda) a(\lambda) \quad \text{[2]} $$

where $I_0$ is the extraterrestrial irradiance at the mean earth–sun distance for wavelength $\lambda$, $d_{es}$ is the dimensionless correction factor for the earth–sun distance, and $\tau$ is the dimensionless transmittance of the atmosphere. The subscripts $o$, $w$, $m$, $R$, and $a$ denote ozone absorption, water vapor absorption, uniformly mixed gas absorption, molecular Rayleigh scattering, and aerosol attenuation, respectively. Clear-sky diffuse irradiance on a horizontal surface consists of a Rayleigh scattering component $I_{R0}$ an
aerosol scattering component $I_{\text{sd}}$, and a component that accounts for multiple reflection between the ground and the air $I_{\text{g}}$ (Bird and Riordan, 1986):

$$I_{\text{g}} (\lambda) = I_{\text{g}} (\lambda) d_a \cos(\theta) \tau_x (\lambda) \tau_{\text{aw}} (\lambda) \tau_{\text{as}} (\lambda) (1 - \tau_{\text{w}}^0 (\lambda)) 0.5 \right \}$$

$$I_{\text{s}} (\lambda) = I_{\text{s}} (\lambda) d_a \cos(\theta) \tau_x (\lambda) \tau_{\text{aw}} (\lambda) \tau_{\text{as}} (\lambda) (1 - \tau_{\text{w}}^0 (\lambda)) 0.5 \right \}$$

$$P_{\text{g}}^0 (\lambda) = \frac{I_{\text{dir}^0} (\lambda) \cos(\theta) \alpha_{\text{sky}} (\lambda) \sigma_{\text{g}}^0 (\lambda)}{1 - \alpha_{\text{sky}} (\lambda) \sigma_{\text{g}}^0 (\lambda)}$$

$$I_{\text{g}} (\lambda) = \frac{I_{\text{dir}} (\lambda) + I_{\text{g}} (\lambda) \alpha_{\text{sky}} (\lambda) \sigma_{\text{g}} (\lambda)}{1 - \alpha_{\text{sky}} (\lambda) \sigma_{\text{g}} (\lambda)}$$

where $\theta$ is the solar zenith angle, $F_{\text{sd}}$ is the fraction of aerosol scatter that is directed downward, and $\alpha$ is the albedo. Transmittance subscripts $\text{sky}$ and $\text{g}$ denote aerosol absorption and aerosol scattering, respectively. Albedo subscripts $\text{sky}$ and $\text{g}$ denote sky reflectivity and ground reflectivity, respectively. The overbar indicates that an areal average albedo value needs to be used. Superscript $\mu$, representing the cosine of the solar zenith angle (horizontal terrain) or the cosine of the illumination angle (sloping terrain), is used to denote direct (beam) radiation. The factor 0.5 is based on the assumption that one-half of the Rayleigh scatter is directed downward. The factors 0.95 and 1.5 are empirical correction factors to account for the fact that Rayleigh and aerosol scattering are not entirely independent of each other. The sum of $I_{\text{R}}$, $I_{\text{sd}}$, and $I_{\text{g}}$ is further corrected by multiplying by $(\lambda + 0.55)^{1.8}$ for $\lambda \leq 0.45 \mu \text{m}$ (for further details, see Bird and Riordan, 1986). Additional information on the calculation of spectral atmospheric transmittances can be found in Dozier and Riordan (1986). Furthermore, for calculating diffuse irradiance on a horizontal plane $I_{\text{dir}}$, a function of cloud cover $c$ (Munro and Young, 1982):

$$I_{\text{dir}} (\lambda) = I_{\text{dir}^0} (\lambda) \cos(\theta) (1 - c)$$

Similarly, diffuse irradiance from the cloudless portion of the sky on a horizontal surface $I_{\text{dif1}}$ is

$$I_{\text{dif1}} (\lambda) = I_{\text{R}}(\lambda) + I_{\text{dir}} (\lambda) + P_{\text{g}}^0 (\lambda) + I_{\text{g}} (\lambda) (\lambda + 0.55)^{1.8} (1 - c)$$

$$\lambda \leq 0.45 \mu \text{m}$$

$$I_{\text{dif1}} (\lambda) = I_{\text{R}}(\lambda) + I_{\text{dir}} (\lambda) + P_{\text{g}}^0 (\lambda) + I_{\text{g}} (\lambda) (1 - c)$$

$$\lambda > 0.45 \mu \text{m}$$

Calculation of the diffuse irradiance from the cloudy portion of the sky $I_{\text{dif2}}$ is complicated and the subject of ongoing research. Our method (horizontal surface) is a rough approximation based on the work of Munro and Young (1982):

$$I_{\text{dif2}} (\lambda) = c I_{\text{g}} (\lambda) d_a \cos(\theta) \tau_x (\lambda) \tau_{\text{aw}} (\lambda) \tau_{\text{as}} (\lambda) (1 - \alpha_{\text{as}} - \beta_{\text{dir}}) 0$$

where the cloud-top albedo $\alpha_{\text{ct}}$ is calculated through a modified expression developed by Fritz (1954) for clouds with large drops (Munro and Young, 1982). The factor $\beta_{\text{sd}}$ denotes the dimensionless absorptivity of clouds. We adopted $\beta_{\text{sd}} = 0.2$ based on evidence presented by Ackerman et al. (2003). Note that the transmittance term for water vapor absorption, $\tau_{\text{w}} (\lambda)$, is not included in Eq. [6]. Stephens (1996) noted that cloud absorption occurs in place of, rather than in addition to, clear-sky water vapor absorption. The spectral effects of clouds on solar irradiance are well understood (Bartlett et al., 1998), hence $\alpha_{\text{ct}}$ and $\beta_{\text{cl}}$ are assumed to be independent of wavelength.

Additional diffuse irradiance is due to multiple scattering between the cloud base and the ground. For a horizontal surface,

$$P_{\text{dif3}}^0 (\lambda) = \frac{c I_{\text{g}} (\lambda) + I_{\text{g}} (\lambda) \alpha_{\text{cb}} \sigma_{\text{g}}^0 (\lambda)}{1 - \alpha_{\text{cb}} \sigma_{\text{g}}^0 (\lambda)}$$

$$I_{\text{dif3}} (\lambda) = \frac{c I_{\text{g}} (\lambda) + I_{\text{g}} (\lambda) \alpha_{\text{cb}} \sigma_{\text{g}} (\lambda)}{1 - \alpha_{\text{cb}} \sigma_{\text{g}} (\lambda)}$$

where $\alpha_{\text{cb}}$ is the cloud-base albedo, which is assumed to be independent of wavelength. Davies et al. (1975) reported $\alpha_{\text{cb}}$ values ranging from 0.2 for cirrus clouds to 0.66 for nimbostratus clouds. Following Munro and Young (1982), a constant $\alpha_{\text{cb}}$ value of 0.6 was selected. The total all-sky diffuse irradiance on a horizontal surface $I_{\text{dif}}$ is now

$$I_{\text{dif}} (\lambda) = I_{\text{dif1}} (\lambda) + I_{\text{dif2}} (\lambda) + P_{\text{dif3}}^0 (\lambda) + I_{\text{dif3}} (\lambda)$$

Terrain slope and aspect may have a significant impact on the actual irradiance received by a surface. All-sky direct irradiance on a sloping plane can be calculated by considering the incidence angle $Z$, which is the angle between the surface normal and the direction of the sun (Munee, 1997):

$$I_{\text{dir}} (\lambda) = I_{\text{dir}^0} (\lambda) \cos(Z) (1 - c)$$

The simplest model for calculating diffuse irradiance on a sloping plane assumes an isotropic sky, resulting in a correction factor of $\cos^2(\beta_{\text{sd}})$, where $\beta$ is the slope angle of the surface plane. Diffuse irradiation is not isotropic in nature, however, and is also a function of the solar zenith angle and the aspect of the slope. Hay (1979) developed a relatively simple model that differentiates between circumsolar and uniform background sky-diffuse components. For all-sky conditions, Hay’s model can be written as (Munee, 1997):

$$I_{\text{dir}} (\lambda) = I_{\text{dir}^0} (\lambda) \left( \frac{\tau_{\text{dir}} (1 - c) \cos(Z)}{\cos(\theta)} + [1 - \tau_{\text{dir}} (1 - c) \cos(\theta)] \cos^2 \left( \frac{i}{2} \right) \right)$$

where $\tau_{\text{dir}}$ is the direct irradiance transmittance of the atmosphere ($\tau_{\text{dir}}(\lambda) \tau_{\text{aw}} (\lambda) \tau_{\text{as}} (\lambda)$). This equation predicts relatively high circumsolar irradiance for clear-sky conditions, and relatively high sky-diffuse irradiance for overcast sky conditions. Horizon brightening is not included in this model. Summation of the spectral irradiance estimates for the visible (<0.7 $\mu \text{m}$) and near-infrared (0.7 $\mu \text{m}$) wavebands completes the incoming direct and diffuse shortwave radiation calculation. The ground surface albedo for soil without
snow is estimated following Dickinson et al. (1993) and Bonan (1996) from soil color class, topsoil water content, and wavelength. The ground surface albedo for snow-covered surfaces is estimated following Marshall (1989) and Bonan (1996) as a function of snow soot content, snow grain radius, wavelength, and illumination angle. Snow albedo decreases as the illumination angle decreases, the soot content increases, and the snow grain diameter increases (Bonan, 1996). Incoming longwave radiation from the sky is calculated as

\[ L_{\text{sky}} = \varepsilon_a \sigma (T_a + 273.15)^4 \cos^2 \left( \frac{i}{2} \right) \]  

where \( \sigma \) is the Stefan–Boltzmann constant and \( \varepsilon_a \) is the emissivity of the atmosphere. The emissivity is calculated as (Brutsaert, 1975; Kustas et al., 1994)

\[ \varepsilon_a = 1.72 \left( \frac{P_a}{T_a + 273.15} \right)^{1/7} \left( 1 + 0.2c^2 \right) \]  

where \( P_a \) is the vapor pressure in the atmosphere (in kPa). Brutsaert’s emissivity calculation assumes a standard atmosphere, which is incorrect for higher elevations where the air is relatively thin. We adopted the correction scheme of Marks and Dozier (1979) to estimate an effective emissivity that is realistic for mountainous areas.

Parameterization of Vegetation

Vegetation is characterized by specifying the vegetation height \( z_v \), the leaf area index (LAI), stem area index (SAI), and soil cover, SC. The exposed LAI and SAI, and the effective soil cover (SC_e), are calculated as a function of the snow height, \( z_{in} \):

\[ \text{LAI}_e = \text{LAI} \left( 1 - \frac{z_{in}}{z_v} \right) \]  

\[ \text{SAI}_e = \text{SAI} \left( 1 - \frac{z_{in}}{z_v} \right) \]  

\[ \text{SC}_e = \begin{cases} \text{SC} & z_{in} < z_v \\ 0 & z_{in} \geq z_v \end{cases} \]

The interception rate by vegetation, \( q_{\text{int}} \), does not distinguish between liquid and solid phases (Oleson et al., 2004):

\[ q_{\text{int}} = \left( p_i + p_{in} \right) \left\{ 1 - \exp \left( -0.5(\text{LAI}_e + \text{SAI}_e) \right) \right\} \]  

The maximum amount of water that the canopy can hold, \( W_{\text{max}} \) (in m) is estimated as (Dickinson et al., 1993)

\[ W_{\text{max}} = 1 \times 10^{-4} (\text{LAI}_e + \text{SAI}_e) \]  

The wetted fraction of the canopy (stems plus leaves), \( F_{\text{wet}} \) is estimated as (Deardorff, 1978; Dickinson et al., 1993)

\[ F_{\text{wet}} = \left( \frac{W}{W_{\text{max}}} \right)^{2/3} \leq 1 \]  

where \( W \) is the amount of intercepted water stored on the canopy. The factors 0.5 (Eq. [14]), \( 1 \times 10^{-4} \) m (Eq. [15]), and \( 2/3 \) (Eq. [16]) are default empirical values that can be optimized if detailed interception data are available.

Canopy Energy Balance

The canopy is assumed to have zero heat capacity. It is also assumed that photosynthetic and respiratory energy transformations can be neglected. This results in the following canopy energy balance equation (Oleson et al., 2004):

\[ I_{nc} + L_{nc} (T_c) - H_c (T_c) - Q_{ed} (T_c) - Q_{tw} (T_c) = 0 \]  

where \( I_{nc} \) is the solar radiation absorbed by the vegetation, \( L_{nc} \) is the longwave radiation absorbed by the vegetation, \( H_c \) is the sensible heat flux from the vegetation, \( Q_{ed} \) is the latent heat flux from the dry fraction of the canopy (transpiration), and \( Q_{tw} \) is the latent heat flux from the wet fraction of the canopy (evaporation of intercepted water). All the energy fluxes except \( I_{nc} \) are a function of the canopy temperature, \( T_c \). The energy balance is solved by finding the correct value for \( T_c \) using Newton–Raphson iteration. The expressions used for the individual energy balance terms in Eq. [17] are listed in Appendix A.

The above canopy energy balance calculation uses the surface temperature and the soil moisture status from the previous time step. This simplification reduces the computational burden because it eliminates the need for an iterative solution between the canopy energy balance, the surface energy balance, and the belowground water flow and heat transport calculations. The associated error in the overall energy balance can be minimized by selecting small time steps.

Surface Energy Balance

The ground surface can be either soil or snow. Fresh snow is incorporated at the beginning of the time step. The surface energy balance for each time step is written as

\[ Q_{g} (T_g) = Q_t + I_{ng} + I_{ng} (T_g) - H_g (T_g) - Q_e \]  

where \( Q_{g} \) is the conductive heat flux between the soil or snow subsurface and the surface as calculated by Fourier’s equation, \( Q_t \) is the advected heat from rainfall, \( I_{ng} \) is the net incoming short-wave radiation, \( I_{ng} \) is the net incoming longwave radiation, \( H_g \) is the outgoing sensible heat flux, and \( Q_e \) is the outgoing latent heat flux due to evaporation and condensation. The equations used for the individual energy balance terms in Eq. [18] are given in Appendix B.

The surface energy balance is solved by calculating the surface temperature \( T_g \) using Newton–Raphson iteration. The conductive heat flux \( Q_{g} \) is calculated using near-surface soil or snow temperatures from the previous time step. In addition, the surface vapor pressure that is used to calculate the latent heat flux \( Q_e \) for soil surfaces without snow is obtained using the near-surface soil water pressure head and the near-surface soil temperature from the previous time step. This simplification reduces the computational burden in a similar way as for the canopy energy balance.

Snow Water Flow and Snow Physical Properties

Snow is described using a multilayer approach to allow simulation of the often nonlinear temperature profile in this medium. Thin snow layers that drop below a preset minimum thickness are
merged with an underlying layer (overlying layer in case of the bottom snow layer). Snow layers that exceed a preset maximum thickness are split into equal parts. Snow water flow and storage are calculated using

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial q}{\partial z}$$  \[19\]

where $\theta_w$ is the volumetric (liquid) water content, $t$ is time, $q$ is the vertical water flux, and $z$ is the vertical coordinate. The water flux in snow is assumed to be driven by gravity only and is estimated as (Colbeck and Davidson, 1973)

$$q = K_{sn} \left( \frac{\theta_w - \theta_r}{1 - \theta_i - \theta_r} \right)^3$$  \[20\]

where $K_{sn}$ is the saturated hydraulic conductivity of the snow, $\theta_i$ is the volumetric ice content, and $\theta_r$ is the residual water content. Equations [19] and [20] are solved sequentially using the old $\theta_w$ to calculate $q$ (Eq. [20]), which is then used to update $\theta_w$ (Eq. [19]). For snow, the residual water content is calculated as (Tarboton and Luce, 1996)

$$\theta_r = \frac{F_i \rho_{sn}}{\rho_w}$$  \[21\]

where $F_i$ is the mass of water that can be retained per mass of dry snow ($=0.02$), $\rho_{sn}$ is the density of snow, and $\rho_w$ is the density of water. The saturated hydraulic conductivity of snow is calculated from the snow grain diameter, $d_{gr}$, and $\rho_{sn}$ using (Shimizu, 1970; Male and Gray, 1981; Jordan, 1991)

$$K_{sn} = 0.077 \frac{\rho_w g}{\eta} d_{gr}^2 \exp \left(\frac{-7.8 \rho_{sn}}{\rho_w}\right)$$  \[22\]

where $\eta$ is the viscosity of water, $g$ is the acceleration due to gravity, and 0.077 and 7.8 are dimensionless empirical parameters. The calculation of $d_{gr}$ is based on the US. Army Corps of Engineers SNTHERM.89 model (Jordan, 1991; snow.usace.army.mil/model_info/sntherm.html [verified 22 July 2009]). Changes in snow diameter in dry snow are primarily due to upward-moving vapor flux. This process is approximated using (Jordan, 1991)

$$\frac{\partial d_{gr}}{\partial t} = \frac{5 \times 10^{-7}}{d_{gr}} D_c \left( \frac{100}{P_a} \right) \left( \frac{T + 273.15}{273.15} \right)^6 \rho_T \frac{\partial T}{\partial z}$$  \[23\]

where $D_c$ is the effective diffusion coefficient for water vapor in snow at 100 kPa and 0°C (0.92 × 10⁻⁴ m² s⁻¹), $P_a$ is the atmospheric pressure (in kPa), and $\rho_T$ is the variation of saturation vapor density with temperature. The units for the factors $5 \times 10^{-7}$ and 100 are m³ kg⁻¹ and kPa, respectively. There is a marked increase in grain growth for wet snow (Colbeck, 1982). Jordan (1991) approximated this process as

$$\frac{d(d_{gr})}{dt} = \begin{cases} 4 \times 10^{-12} \left( \theta_w + 0.05 \right) & 0 < \theta_w < 0.09 \\ 4 \times 10^{-12} (0.14) & \theta_w \geq 0.09 \end{cases}$$  \[24\]

where the unit for the factor $4 \times 10^{-12}$ is m² s⁻¹. The snow compaction rate, CR, for each layer is calculated using (Jordan, 1991)

$$CR = \frac{c_1 c_2 c_3}{\eta_0} \exp \left( -c_4 \left(0 - T\right)\right)$$  \[25\]

where $\eta_0$ is the snow overburden (kg m⁻²) and $\eta_0 (= 0.9 \times 10^6$ kg m⁻² s⁻¹) is a viscosity coefficient. The first part of the equation describes compaction due to snow metamorphism, while the second part describes compaction due to overburden. Recommended values for the constants are: $c_1 = 2.778 \times 10^{-6}$ m² kg⁻¹, $c_2 = 0.04 °C⁻¹$, $c_3 = 0.08 °C⁻¹$, and $c_6 = 0.023$ m³ kg⁻¹. The dimensionless factors $c_2$ and $c_3$ are

$$c_2 = \begin{cases} \exp \left( -0.046(\rho_{sn} - 100) \right) & \rho_{sn} > 100 \text{ kg m}^{-3} \\ 1 & \rho_{sn} \leq 100 \text{ kg m}^{-3} \end{cases}$$  \[26a\]

$$c_3 = \begin{cases} 2 & \theta_w > 0 \\ 1 & \theta_w = 0 \end{cases}$$  \[26b\]

where the factor 0.046 is in m³ kg⁻¹. The compaction rate is used to update the thickness of each snow layer $d$:

$$\frac{d(d)}{dt} = -dCR$$  \[27\]

Finally, the new snow layer thickness is used to update the (liquid) water content $\theta_w$ and ice content $\theta_i$ of each layer. This allows the new snow density to be calculated:

$$\rho_{sn} = \theta_w \rho_w + \theta_i \rho_i$$  \[28\]

where $\rho_i$ is the density of ice.

### Soil Water Flow

Vertical soil water is calculated using a noniterative solution to the Richards equation following a procedure outlined in Ross (2003). The procedure is best explained by showing the numerical discretization. The mass balance for soil layer $i$ can be written as

$$d_i \frac{\Delta \theta_{ui}}{\Delta t} = q_{i+1}^F - q_i^F - S_i d_i$$  \[29\]

where $S$ is a sink term to account for root water uptake. The soil water flux at a fraction $F$ through the time step is estimated using a Taylor series expansion:

$$q_i^F = q_i^0 + F \left( \frac{\partial q_i}{\partial u_i} \Delta u_i + \frac{\partial q_i}{\partial u_{i-1}} \Delta u_{i-1} \right)$$  \[30\]

where $F$ is a dimensionless weighting factor (between 0 and 1), and $u$ is either the volumetric soil water content $\theta_w$ (unsaturated layer) or the soil water pressure head $h$ (saturated layer). The superscript 0 denotes the beginning of the time step. The soil water flux at the beginning of the time step is calculated using the Darcy equation:

$$q_i^0 = K_j + K_{r-1} \left( \frac{h_i - h_{i-1}}{\Delta z_j} + 1 \right)$$  \[31\]
where $K$ is the soil hydraulic conductivity. The derivatives of the soil water flux at the beginning of the time step can be obtained by differentiating the Darcy equation with respect to either $\theta_w$ or $h$. The sink term is calculated as

$$S_i = \frac{a_i(b_i)SC_iQ_{id}}{\sum a_i(b_i)d_i} \rho_{w}\gamma_v$$

where $\alpha$ is the dimensionless root water uptake reduction factor as a function of soil water pressure head according to Feddes et al. (1978). In Eq. [32] it is assumed that all soil layers contribute equally to root water uptake, both below the canopy and in the interspaces areas. The above expressions result in a tridiagonal system of equations that can be solved for $\alpha$ using the Thomas algorithm (Press et al., 1992). The weighting factor $F$ is 0.5 if the entire soil profile is unsaturated to improve accuracy. Otherwise, $F = 1$ is used to improve stability. An additional equation for pond height $h_0$ is included if ponding occurs on the soil surface (Ross, 2003):

$$\frac{\Delta h_0}{\Delta t} = q_{\text{top}}^F - q_{\text{surf}}^F$$

where $q_{\text{top}}$ is the net incoming water flux from precipitation and surface evaporation (no snow) or snowmelt and $q_{\text{surf}}$ is the flux at the soil surface. The surface flux is again estimated using a Taylor series expansion:

$$q_{\text{surf}}^F = q_{\text{surf}}^0 + \left( \frac{\partial q_{\text{surf}}}{\partial h_0} \right)^0 \Delta h_0 + \left( \frac{\partial q_{\text{surf}}}{\partial u_N} \right)^0 \Delta u_N$$

where $N$ is the number of subsurface layers (soil and bedrock, numbering is from the bottom up). The surface flux at the beginning of the time step is

$$q_{\text{surf}}^0 = K_N \left( h_0 - h_{CN} + 1 \right)$$

An adjustable time step is used in the soil water flow calculation so that the maximum change in the volumetric soil water content is 0.02 and the maximum overshoot in the surface ponding layer is ~0.02 m (negative ponding layer). The soil hydraulic properties are described by combining the Brooks and Corey (1964) water retention function with the Mualem (1976) hydraulic conductivity function:

$$\frac{\theta_w - \theta_r}{\phi - \theta_r} = \left( \frac{b}{h_0} \right)^{-\chi}$$

$$K = K_s \left( \frac{\theta_w - \theta_r}{\phi - \theta_r} \right)^{2+2\chi} \times 10^{-2h_i}$$

where $\phi$ is the effective soil porosity, $b_i$ is the bubbling pressure head, $\chi$ is the pore-size distribution index, $K_s$ is the saturated soil hydraulic conductivity, and $h_i$ is the pore connectivity or tortuosity factor. The soil hydraulic conductivity is reduced using an impedance factor $\Omega = 15$ to account for reduced hydraulic conductivity in frozen soils (Hansson et al., 2004). Frozen soils may exhibit steep gradients in soil water pressure heads near the freezing front. Simply averaging the soil hydraulic conductivities of two neighboring cells will overestimate the soil water flow toward the front. Hence, in frozen soil regions, only the cell with the lowest conductivity is used for $K$ in the Darcy flow calculation (Lundin, 1990).

**Snow–Soil–Bedrock Heat Transport**

Heat transport in the snow–soil–bedrock continuum is calculated using the following general equation describing both heat conduction and advection:

$$\frac{\partial (C_vT)}{\partial t} = \frac{\partial }{\partial z} \left[ K_v \frac{\partial T}{\partial z} \right] + C_v \frac{\partial (q_T)}{\partial z} - C_v s ST$$

where $C_v$ is the volumetric heat capacity and $K_v$ is the thermal conductivity. The subscript $w$ denotes liquid water. This equation is solved using an implicit backward difference scheme for maximum numerical stability (Campbell, 1985). The effect of possible ponding layer on the vertical heat transport is ignored in the model. The heat capacity of snow, soil, and bedrock are calculated as

$$C_v = \theta_w C_{v,w} + \theta_s C_{v,s} \quad \text{(snow)}$$

$$C_v = (1 - \theta) C_{v,so} + \theta_s C_{v,s} \quad \text{(soil)}$$

$$C_v = C_{v,r} \quad \text{(bedrock)}$$

where subscripts $i$, $so$, and $r$ indicate ice, soil solids, and rock, respectively. The small contribution of air to the volumetric heat capacity is neglected in the above equations. The calculation of thermal conductivity is less straightforward because the spatial arrangement of the different phases is important. The snow thermal conductivity is estimated from snow density using the following expression (Jordan, 1991):

$$K_s = \kappa_s + \left( 7.75 \times 10^{-5} \rho_{m} + 1.105 \times 10^{-6} \rho_{m}^2 \right) (\kappa_{sat} - \kappa_s)$$

where the subscript $a$ denotes air. The factors $7.75 \times 10^{-5}$ and $1.105 \times 10^{-6}$ have units m$^3$ kg$^{-1}$ and m$^6$ kg$^{-2}$, respectively. The soil thermal conductivity calculation follows Farouki (1981) and references therein:

$$K_v = F_{KN} (\kappa_{sat} - \kappa_{dry}) \quad \text{(soil)}$$

where $F_{KN}$ is the Kersten number and the subscripts $dry$ and $sat$ denote dry soil and saturated soil, respectively. The Kersten number is a function of relative water saturation, with different expressions for frozen and unfrozen soils. Details on the calculation of $F_{KN}$, $\kappa_{dry}$ and $\kappa_{sat}$ can be found in Farouki (1981), Peters-Lidard et al. (1998), and Oleson et al. (2004). The bedrock thermal conductivity is represented by a single value, based on the rock mineral composition (e.g., Clauser and Huenges, 1995):

$$K_v = K_{r} \quad \text{(bedrock)}$$

No advective heat transport is calculated in the bedrock ($q = 0$). Deep percolation from the bottom of the soil profile is simply removed from the model. This water loss is the result
of downward fracture flow or lateral subsurface flow across the soil–bedrock interface.

Snow and Soil Water Phase Change

Liquid water–ice phase change in a snow layer depends on the layer temperature and on the net incoming heat flux. In soil, the energy state of the liquid water also plays a role. Capillary forces and dissolved ions reduce the energy state of the soil water, resulting in below 0°C freezing temperatures. The rate of phase change is determined by the total available energy, $Q_{pc}$, estimated as (e.g., Oleson et al., 2004)

$$Q_{pc} = \frac{\Delta (C,T)}{\Delta t} - \frac{C_{v,old}}{\Delta t} \frac{0 - T_{old}}{\Delta t}$$

where the first term to the right of the equal sign constitutes the net incoming energy and the second term constitutes the energy storage in the layer relative to the freezing point. The net incoming energy is based on end-of-time-step values, while the relative energy storage is based on start-of-time-step values (subscript old).

For snow, the changes in ice content, water content, and temperature due to freeze–melt can be calculated using $Q_{pc}$ provided that enough liquid water is present to freeze ($T < 0$, $Q_{pc} < 0$) and enough ice is present to melt ($T \geq 0$, $Q_{pc} > 0$):

$$\frac{\Delta \theta_{f}}{\Delta t} = -\frac{Q_{pc}}{\rho_{f} \gamma_{f}}$$

$$\frac{\Delta \theta_{w}}{\Delta t} = \frac{Q_{pc}}{\rho_{w} \gamma_{f}}$$

$$\frac{\Delta T}{\Delta t} = -\frac{Q_{pc}}{C_{v}}$$

where $\gamma_{f}$ is the latent heat of fusion ($\sim 333.5$ kJ kg$^{-1}$). The same equations can be used to calculate changes in ice content, water content, and temperature in the soil due to freeze–thaw, with one additional condition. Freezing in the soil can only occur when the water potential due to capillary forces and dissolved ions is higher than the equilibrium potential of liquid water in contact with ice (Spaans and Baker, 1996; Koren et al., 1999). The equilibrium potential of liquid water in contact with ice ($h_{eq}$ in m, $T \leq 0°C$) is calculated by integrating the Clapeyron equation, assuming zero ice pressure (Fuchs et al., 1978; Spaans and Baker, 1996):

$$h_{eq} = \gamma_{f} \ln \left[ \frac{(T + 273.15)}{273.15} \right]$$

Soil water freezing now requires that $h + h_{osm} > h_{eq}$ where $h_{osm}$ is the osmotic head due to dissolved ions. The osmotic head is calculated as

$$h_{osm} = -\left( \frac{\phi}{\theta_{w}} \right) \frac{R (T + 273.15) m'}{g}$$

where $R$ is the gas constant (8.3 J mol$^{-1}$ K$^{-1}$) and $m'$ is the molality (mol solute kg$^{-1}$ water). No solute transport is incorporated in the model and a constant molality of $1.34 \times 10^{-2}$ mol kg$^{-1}$ is assumed.
optical depth was assumed in summer to account for the higher atmospheric dust concentrations during this period. The water vapor amount used to calculate the contribution of water vapor to the transmittance was estimated from the vapor pressure and the atmospheric pressure using the empirical relationship of Garrison and Adler (1990). Transmittances due to uniformly mixed gas absorption and molecular Rayleigh scattering followed Bird and Riordan (1986), with all coefficients remaining unchanged.

Meteorological input was taken from the small meteorological station at the study site. Relative humidity, air temperature, wind speed, and precipitation were specified at 15-min intervals. No observations of cloud cover were available. Instead, cloud cover was estimated using the solar radiation data. This was achieved by first identifying clear sky days and by fitting a simple power law equation of the form $I_{\text{tot}} = b_1d_{\text{gs}}(\cos \theta)^{b_2}$ to the observed total solar radiation $I_{\text{tot}}$ during these days (Long and Ackerman, 2000). This yielded $b_1 = 1093.6$ J m$^{-2}$ s$^{-1}$ and $b_2 = 1.2$ for our study site. Subsequent comparison of the power law (clear sky) $I_{\text{tot}}$ for a given daytime period to the observed $I_{\text{tot}}$ for that period allowed us to identify cloudy ($c = 1$) and uncloudy ($c = 0$) episodes. Nighttime cloudiness was estimated by averaging the cloudiness during the final 2 h of the preceding afternoon and the first 2 h of the following morning.

The vegetation height $z_v$ was taken to be 0.4 m, based on the average height of the sagebrush at the soil moisture sensor site. Soil cover at the site during the summer growing season was estimated at 0.55 by Williams (2005). The maximum LAI, minimum LAI, and SAI of a single average plant at the site were estimated at 2.3, 0.2, and 0.2, respectively. In principle, bare areas and vegetated areas could be treated separately by the model; however, this is probably not appropriate when the bare and vegetated sites are closely interspersed such as at our site. Instead, we chose to consider the entire site vegetated (SC = 1), with an adjusted maximum LAI, minimum LAI, and SAI of 1.265, 0.11, and 0.11, respectively (single-plant values multiplied by 0.55). The actual LAI was assumed to be a function of the depth-average soil temperature (Dickinson et al., 1993):

$$\text{LAI} = \text{LAI}_{\text{min}} + (\text{LAI}_{\text{max}} - \text{LAI}_{\text{min}}) \left[ 1 - 0.0016 \left( 25 - T_{so} \right)^2 \right]$$

where $T_{so}$ is the soil temperature. Plant optical properties and plant aerodynamic parameters used in the canopy energy balance calculation were represented by parameters for the “broadleaf evergreen shrub—temperate” plant functional type as given by Oleson et al. (2004). It was assumed that the entire soil profile contributed equally to the potential root water uptake. The following root water uptake reduction factors were assumed: no reduction for soil water pressure heads between −7 and −0.01 m, and linear reduction to zero uptake for pressure heads between −7 and −160 m and for pressure heads between −0.01 and 0 m.

The density of fresh snow was calculated as a function of air temperature according to the empirical relationship of Anderson (1976). The grain diameter of fresh snow was assumed to be 0.05 mm. Fresh snow on top of existing snow was incorporated into the top snow layer by averaging the properties. This procedure minimized numerical instabilities in the heat transport calculations associated with thin snow layers. Snow layers thinner than 0.05 m were merged with neighboring layers, if present. Snow layers thicker than 0.2 m were split into two equal parts. Snow albedo, according to the Marshall (1989) and Bonan (1996) equations used, was a function of snow soot concentration, among others. The increase of soot concentration $s$ (g g$^{-1}$) with time after deposition $t_d$ was approximated in this study as

$$s = \frac{t_d}{t_{d,\text{max}}}(t_{\text{max}} - t_{\text{min}}) + t_{\text{min}} \leq t_{\text{max}}$$

where $t_{d,\text{max}}$ is taken to be 20 d, $t_{\text{min}}$ is $3.5 \times 10^{-8}$ g g$^{-1}$, and $t_{\text{max}}$ is $1 \times 10^{-6}$ g g$^{-1}$ (e.g., Hansen and Nazarenko, 2004). The land surface albedo for snowpacks thinner than 0.1 m was calculated by weighting the snow albedo and the soil albedo, assuming exponential extinction of the radiation penetration of the snow (see Tarboton and Luce, 1996).

The 1.25-m-deep soil at the site was discretized into 14 layers, with thicknesses increasing from 0.025 m at the surface to 0.2 m at the soil–bedrock interface. The underlying bedrock was discretized into 11 layers, with thicknesses increasing from 0.3 m at the soil–bedrock interface to 1.4 m at the bottom of the domain. This resulted in a total subsurface thickness of 10.45 m. The relatively thick subsurface used was important to account for the damping effect of the bedrock heat storage on the seasonal soil temperature variations. The initial soil water content and soil temperature were derived from the reflectometer data and the thermocouple data, respectively. The initial bedrock temperature was unknown. A constant temperature of 8.9°C was assumed at the bottom of the bedrock at 10.45-m depth. This temperature was calculated by averaging the mean annual air temperatures for the calibration (8.4°C) and the validation (9.3°C) periods. Initial bedrock temperatures at shallower depths were approximated by running the model twice, first with estimated initial temperatures and then with initial values derived from the final calculated bedrock temperatures from the first run.

The top boundary for the numerical soil water flow calculations was either the soil surface (no ponding) or the ponded water surface. In both cases, a flux condition was used. This flux was determined by the difference between precipitation and evaporation (no snow) or by the melt flux from the bottom snow layer. The bottom boundary for the soil water flow calculations was the soil–bedrock interface. The exact flow conditions at this interface were difficult to define. Lateral subsurface flow, downward fracture flow, and vertical porous media flow were all probable at this boundary during all or part of the year. We simply used a free-drainage boundary condition; $q = K(\theta_s)$. Note that water flow through the bedrock was not accounted for in the model. Instead, the water flux at the soil–bedrock interface was simply removed from the model and classified as deep percolation.

The top boundary for the numerical heat transport calculation was either the soil surface (no snow) or the snow surface. In both cases, a heat flux was prescribed. This heat flux was determined by the surface energy balance. The possible presence of a ponding layer was ignored in the heat transport calculations. The bottom boundary for the heat transport calculation was set at the bottom of the bedrock. Here a constant temperature of 8.9°C was prescribed. A maximum ponding layer of 2 cm was allowed at the soil surface. Buildup of water in excess of 2 cm was removed from the model and classified as surface runoff.
Model Calibration

No attempt was made to calibrate the canopy energy balance, surface energy balance, or snowpack components of the model. The canopy and surface energy balances could not be verified because of a lack of independent data for checking the model output. The snowpack calculations contain many semitheoretical parameters that could, in principle, be optimized using the snow height, snow water equivalent, and snow density data from the subcatchment. We decided against such a calibration, given the large number of snow parameters, and given the relatively crude snow physical data set available compared with the snow studies from which the default snow parameters were developed. Only detailed snow height data were available to us, supplemented with some snow water equivalent and snow density data. Snow grain size was not measured at all. Therefore, the snow height, snow water equivalent, and snow density data were only used to check the performance of the snow calculations.

Soil hydraulic parameters in the Brooks–Corey–Mualem functions were determined using data from a single multistep outflow experiment on an undisturbed soil sample and by inverse modeling using the CS615 soil water content data from the calibration period. A homogenous soil profile was assumed. The undisturbed sample for the multistep outflow experiment was taken from the southwest-facing slope of the 0.02-km² subcatchment. Initial attempts to estimate all soil hydraulic parameters from the multistep outflow using inverse methods (van Dam et al., 1994; Hopmans et al., 2002) yielded unrealistic parameter estimates due to the limited pressure head range of 0 to −150 cm covered by the outflow experiment. Determining the soil hydraulic parameters by inversely modeling the calibration period using the CS615 soil water content data with the global parameter optimization software MCS (Huyer and Neumaier, 1999) also resulted in unrealistic parameter values. The failure to obtain realistic parameter values using either method was attributed to insufficient information content in the fitting data.

Instead, a three-step calibration approach was used to determine the hydraulic properties of the soil profile. First, the van Genuchten (1980) water retention function was fitted to the pressure head–soil water content data from the outflow experiment using the RETC software (van Genuchten et al., 1991) with $\theta_r = 0.01$. The residual water content $\theta_r$ was fixed to a realistic value for relatively coarse-textured soils to mitigate the fact that the multistep outflow experiment did not cover the dry soil range. Second, the van Genuchten parameters $\alpha_{vg}$ and $n_{vg}$ were used to calculate $h_b = -1/\alpha_{vg}$ and $\chi = n_{vg} - 1$ to obtain the Brooks–Corey water retention parameters. Third, the optimum $K_s$ value was determined by inversely modeling the calibration period using the model coupled to the MCS software with $l = 1$ and $K_s \leq 320$ cm d⁻¹ and with the CS615 water content data in the objective function. The upper limit of $K_s$ of 320 cm d⁻¹ was based on results from falling-head experiments on the subcatchment soils (Gribb et al., 2009). The falling-head $K_s$ values were expected to be relatively high because, under ponded conditions, both the soil matrix and macropores contribute to flow. Surface ponding has never been observed on the northeast-facing slope, allowing us to neglect macropores in the model.

Soil heat transport parameters were not calibrated. Instead, default parameter values were taken from the literature (Clauser and Huenges, 1995; Scharli and Rybach, 2001; Oleson et al., 2004). The specific heats of air, water, ice, and rock were 1.0, 4.2, 2.1, and 0.79 J g⁻¹ K⁻¹, respectively. Volumetric heat capacities for water, ice, and rock were calculated by multiplying the specific heats with the respective densities of 1, 0.92, and 2.7 g cm⁻³. The volumetric heat capacity for air was taken as zero because of the low density of air. The thermal conductivity of air, water, ice, and rock was 0.023, 0.57, 2.29, and 3.25 J m⁻¹ s⁻¹ K⁻¹, respectively. The volumetric heat capacity and thermal conductivity of soil solids were calculated from sand and clay contents using empirical equations provided by Oleson et al. (2004). A sand content of 74% and a clay content of 9% were determined using the hydrometer method on soil samples from the subcatchment (Williams, 2005).

The model calibration and validation were evaluated using graphical comparisons and modeling statistics. Two generally recommended statistical model indicators were used: root mean square error (RMSE) and modeling efficiency (EF) (Loague and Green, 1991; Vanclooster et al., 2000; Fernandez et al., 2002). The RMSE statistic gives the overestimation or underestimation percentage of the predicted value compared with the mean observed value. The EF statistic indicates the degree to which the predictions give a better estimate of the observations compared with the mean of the observations (Fernandez et al., 2002). The maximum value for EF is 1. If EF is <0, the model-predicted values are worse than simply using the observed mean (Loague and Green, 1991). The RMSE and EF values were calculated for snow height (snow sensor), depthwise soil water content, and depthwise soil temperature. No modeling statistics were calculated for snow water equivalent and snow density due to the sparse data set for these parameters.

Results and Discussion

Model Calibration Results

Measured and calculated snow height, snow water equivalent, and snow density for the calibration period at the snow sensor location near Pit 100 on the northeast-facing slope are shown in Fig. 1. The modeling statistics for snow height are given in Table 1. The timing of snow accumulation and snowmelt are captured reasonably well by the model (RMSE = 19%, EF = 0.86). Sharp decreases in the sensor-measured snow height in early February and early March are underestimated by the model. The calculated meltdown of the main snowpack in March is delayed by about 7 d. Sensor-measured and manually measured snow heights can differ significantly, showing the effect of spatial variability across short distances. The snow water equivalent is overestimated by the model by as much as 60%. The calculated average snow density is accurate in February but too high in March. The sharp calculated peaks in snow density at the start of the snow season and at the end of the snow season are due to thin snow layers that rapidly ripen and melt.

Overall, the model seems to simulate the snowpack realistically, with perhaps a small underestimation of the snow ripening and snowmelt rates. A perfect match between measured and calculated physical properties is not expected for a complicated medium such as snow. This is especially true considering the fact that the effect of wind on snow transport is not considered in the model. Snow drifting across the landscape may modify the spatial
distribution of snow considerably, especially when vegetation is present (Essery et al., 1999; Prasad et al., 2001).

The fit between the measured (multistep outflow experiment) and calculated (RETC optimized [van Genuchten, 1980] soil water retention function) is shown in Fig. 2. The retention function with \( \theta_r = 0.01 \), saturated volumetric soil water content \( \theta_s = 0.339 \) (\( \sim \) porosity \( \phi \)), van Genuchten parameters \( \alpha_{vg} = 0.0344 \) cm\(^{-1} \) and \( n_{vg} = 1.297 \) fits the data well (\( R^2 = 0.991 \)). The values for the individual parameters seem realistic for relatively coarse-textured soils (e.g., Carsel and Parrish, 1988). Figure 2 confirms visually that the measurements are clustered in the wet soil water range, which led us to fix \( \theta_r = 0.01 \) in this case.

Note that we did not fit the Brooks and Corey (1964) soil water retention function directly to the data. This function is incapable of producing good fits to soil water retention data in the wet soil range because of the fact that \( \theta_w = \phi \) for \( h \geq h_b \).

Instead, we calculated the Brooks and Corey \( h_b = -1/\alpha_{vg} = -29.1 \) cm and \( \chi = n_{vg} - 1 = 0.297 \) from the van Genuchten function. The relatively poor performance of the Brooks and Corey function in the wet range is not a major concern for the soil water flow calculations presented in this study because the soils rarely approach saturation.

The MCS-optimized value of \( K_s = 38.67 \) cm d\(^{-1} \) is well below the falling-head method \( K_s \) of 320 cm d\(^{-1} \), as expected. The measured and calculated soil water content and soil temperature with depth for Pit 100 on the northeast-facing slope of the subcatchment. Snow height was measured with a distance sensor (solid line) and manually (symbols). Snow water equivalent and snow density are for the entire snowpack.

The value of the residual volumetric soil water content \( \theta_r \) was not optimized but fixed to 0.01.

The underestimation of the calculated soil water content at depths of 30, 60, and 100 cm during the snowpack period (November–March) may be attributed to three causes. First, the underestimation of snow ripening and snowmelt mentioned above may result in an underestimation of infiltrating meltwater during the snow period, resulting in lower than expected soil water contents. Second, the assumption of a homogeneous soil profile may be too simplistic. For example, field observations have found an illuvial clay layer of variable depth at the soil–bedrock interface in the subcatchment (Williams et al., 2008). Third, the free-drainage bottom boundary condition for soil water

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Snow height RMSE</th>
<th>Soil water content RMSE</th>
<th>Soil temperature RMSE</th>
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Table 1. Model statistics for snow height, depthwise soil water content, and depthwise soil temperature for the calibration period. Both root mean square error (RMSE) and modeling efficiency (EF) are given.

Fig. 1. Measured and calculated snow height, snow water equivalent, and snow density for the calibration period at the snow sensor location near Pit 100 on the northeast-facing slope of the subcatchment. Snow height was measured with a distance sensor (solid line) and manually (symbols). Snow water equivalent and snow density are for the entire snowpack.

Fig. 2. Measured and calculated soil water retention. Calculated values were obtained by fitting the van Genuchten (1980) soil water retention function to retention data from a multistep outflow experiment. Optimized soil water retention parameters: saturated volumetric soil water content \( \theta_s = 0.339 \) (\( \sim \) porosity \( \phi \)), van Genuchten parameters \( \alpha_{vg} = 0.0344 \) cm\(^{-1} \) and \( n_{vg} = 1.297 \).

The inclusion of a litter layer in the model that shields the topsoil from the atmosphere might reduce the apparent overestimation of soil water freezing (Flerchinger, 2000).
flow is a gross simplification of the flow conditions at the soil–bedrock interface. Lateral inflow and outflow (e.g., McNamara et al., 2005), downward fracture flow (e.g., Miller et al., 2008), and vertical porous media flow are all possible at the interface. In fact, it is probable that the free-drainage boundary condition overestimated the downward water flow from the soil profile.

The measured and calculated soil temperatures at different depths agree well (13 < RMSE < 28%, 0.93 < EF < 0.97). The moderating effect of the snowpack from November to March on the temporal soil temperature fluctuations is clearly visible. The model underestimates the daily maximum temperature at the 5-cm depth. This may be due to the spatial discretization. The element thickness at this depth was 5 cm, while the thermocouple represents a point measurement. Note that the measured and calculated soil temperature at the 5-cm depth never falls below zero. Both the soil water freezing process during cold periods without snow, and the snowpack during periods with snow, prevent the temperature from falling below the freezing point. The good match between measured and calculated soil temperatures provides indirect evidence that the canopy and surface energy balance calculations as well as the snow and soil heat transport parameters are realistic.

**Model Validation Results**

The measured and calculated snow heights for the validation period at the snow sensor location near Pit 100 are shown in Fig. 4. The modeling statistics for snow height are given in Table 2. Snow accumulation at the onset of winter during November and December is captured accurately by the model. The midwinter (January) snow height is underestimated by about 25 to 40%. Unfortunately, due to equipment failure, no snow height data are available to check the calculated snow height during the melt season in March. The snowpack had completely melted by the time the equipment was back online in April.

The measured and calculated soil water content and soil temperature with depth for Pit 100 for the validation period are shown in Fig. 5. Modeling statistics are given in Table 2. No measurements are available during the first half of February and during most of March due to equipment failure. In addition, the soil water content sensor at the 30-cm depth did not function during the entire year. Note that the measured seasonal soil water content fluctuations during the validation period differ from the measured seasonal fluctuations during the calibration period. This is mainly due to significant rainfall during the second half of May. Rainfall between 10 and 28 May totaled 126 mm, far above normal values for this period.

**Fig. 3.** Calibration period measured and calculated soil water content and soil temperature with depth for Pit 100 on the northeast-facing slope of the subcatchment.

**Fig. 4.** Measured and calculated snow height for the validation period at the snow sensor location near Pit 100 on the northeast-facing slope of the subcatchment. Snow height was measured with a distance sensor.

**Table 2.** Model statistics for snow height, depthwise soil water content, and depthwise soil temperature for the validation period. Both root mean square error (RMSE) and modeling efficiency (EF) are given. No measured soil water content data at 30-cm depth were available (NA).

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Snow height RMSE</th>
<th>Snow height EF</th>
<th>Soil water content RMSE</th>
<th>Soil water content EF</th>
<th>Soil temperature RMSE</th>
<th>Soil temperature EF</th>
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<tr>
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<td>NA</td>
<td>26</td>
<td>0.78</td>
<td>8</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Overall, the measured and calculated soil water contents compare well (20 < RMSE < 38%, 0.65 < EF < 0.86). The soil water contents at the 5-cm depth are overestimated, while the water contents at 60- and 100-cm depths are underestimated by the model. It is especially encouraging that the May rainfall period is simulated reasonably well by the model. The surface water input is not complicated by snowmelt during this period, allowing a more straightforward evaluation of the performance of the calibrated soil hydraulic properties.

The comparison between measured and calculated soil temperature is good for all depths (8 < RMSE < 28%, 0.88 < EF < 0.97). As before, the daily maximum temperatures at 5-cm depth are underestimated by the model. Note that measured and calculated soil temperatures at the 5-cm depth fall below the freezing point for several nights during 1 to 7 November because of the lack of significant snow cover. No significant soil water freezing takes place during these nights because the soil is still dry, allowing the temperatures to drop quickly. Based on the snow height, soil water content, and soil temperature comparisons for the validation period, we conclude that the model was properly calibrated.

**Water and Energy Balance**

The yearly water balances for the calibration and validation periods for Pit 100 are summarized in Table 3. The table shows that yearly evapotranspiration is equivalent to 56 to 58% of the yearly precipitation. Similarly, yearly deep percolation is 39 to 43% of the yearly precipitation. Both the calibration period and the validation period show a small increase of 9 and 22 mm, respectively, in soil water storage during the year. Measured increases in soil water storage according to the CS615 sensors are only 1 and 4 mm for the calibration and validation periods, respectively (values not shown). McNamara et al. (2005) calculated the yearly water balance for Pit 100 and Pit 65 (about 2 m from Pit 100) for approximately the same period as our calibration period using the SHAW model. Their results for yearly evapotranspiration (62% of yearly precipitation) and yearly deep percolation (43% of yearly precipitation) agree with our results.

Williams (2005) used SHAW to calculate the water balance for 57 points throughout the subcatchment for a period that roughly coincides with our validation period. Averaging his results for the three points that surround Pit 100 results in yearly evapotranspiration and deep percolation values that are 68 and 37%, respectively, of yearly precipitation. Both the McNamara et al. (2005) and Williams (2005) results support the validity of our water balance calculations. It should be stressed that the deep percolation term, as used in the above discussion, is interpreted from the viewpoint of the soil profile. At the soil–bedrock interface, the downward percolation will be partitioned into lateral flow, downward fracture flow, and downward porous rock flow, depending on the exact flow conditions at the interface. Lateral unsaturated flow in moist soil above the soil–bedrock interface may also occur given the steep terrain.

The yearly energy balance for the calibration and validation periods for Pit 100 is summarized in Table 4. The calibration period shows an increase in the amount of energy stored in the soil and bedrock of 0.9 MJ m⁻² during the calculation period, while the validation period shows a decrease of 2.7 MJ m⁻². These changes in energy storage are small relative to the total energy stored in the soil and the bedrock (on average 254 MJ m⁻² in

**Table 3. Yearly water balance for the calibration and validation periods for the Pit 100 location on the northeast-facing slope of the subcatchment.**

<table>
<thead>
<tr>
<th>Water balance term</th>
<th>Calibration period</th>
<th>Validation period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precipitation</td>
<td>590</td>
<td>716</td>
</tr>
<tr>
<td>Surface runoff</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Evapotranspiration</td>
<td>328</td>
<td>416</td>
</tr>
<tr>
<td>Deep percolation</td>
<td>253</td>
<td>278</td>
</tr>
<tr>
<td>Change in soil water storage</td>
<td>9</td>
<td>22</td>
</tr>
</tbody>
</table>
late August, data not shown). For the calibration period, the net input of energy comes from both the surface and the subsurface. In contrast, for the validation period, the net input is solely due to the surface and a relatively large amount of energy of 12.4 MJ m\(^{-2}\) yr\(^{-1}\) escapes through the bottom boundary. It is difficult to assess the significance of these results. Lack of bedrock temperature data forced us to estimate the initial bedrock temperatures, including the temperature of 8.9°C that serves as the bottom boundary condition in the heat transport calculations.

Slagstad et al. (2008) stated that the mean annual surface temperature is the main determining factor for shallow (<1000-m depth) bedrock temperatures. The differences in sign for the average bottom heat flux for the calibration period (~0.7 MJ m\(^{-2}\) yr\(^{-1}\)) and the validation period (12.4 MJ m\(^{-2}\) yr\(^{-1}\)) appear to reflect the differences in mean annual air temperatures of 8.4 and 9.3°C, respectively. Changing the initial bedrock temperatures, altering the total thickness of the modeled domain, and modifying the temperature at the bottom boundary, however, will change the energy balance terms, as shown in Table 4. Given the good match between measured and calculated soil temperatures as shown in Fig. 3 and 5, this issue is not further explored here. Possible energy input from the deeper subsurface to the shallow bedrock due to radioactive decay introduces yet another uncertainty to the energy balance calculations.

### Summary and Conclusions

A one-dimensional vertical computer model was developed to quantify the water and energy balance of vegetated areas subject to snow accumulation. The model calculates solar radiation, canopy energy balance, surface energy balance, snowpack dynamics, soil water flow, and snow–soil–bedrock heat exchange, including soil water freezing. The processes are loosely coupled (solved sequentially) to limit the computational burden. Calibration is achieved by optimizing the saturated soil hydraulic conductivity. All other model parameters are based on measurements or default values taken from the literature. Validation results show that the model can successfully calculate snow height, soil water content, and soil temperature for a northeast-facing mountain slope in the Dry Creek Experimental Watershed near Boise, ID.

Water balance results for the calibration and validation periods show that yearly evapotranspiration consumes approximately 60% of the yearly precipitation on the northeast-facing slope. Yearly deep percolation from the soil profile constitutes about 40% of the yearly precipitation. These data confirm earlier results obtained with the SHAW model by McNamara et al. (2005) and Williams (2005). The partitioning of the deep percolation from the soil into lateral flow above the soil–bedrock interface and vertical downward flow into the bedrock is still unclear. This will be the topic of a future study that will quantify spatial patterns in the water flow and heat transport.

This study, for the first time, verifies modeled soil temperatures for the Dry Creek Experimental Watershed. The results are encouraging, with excellent comparisons between measured and calculated soil temperatures. The reliability of the calculated annual energy balance for the calibration and validation periods is difficult to assess because of the lack of depthwise bedrock temperature data. Future measurements of bedrock temperature with time will be helpful in assessing the depth penetration of annual temperature fluctuations that can be used to further constrain the model.

Calibration and validation of the new model is restricted to a single mountain slope in this work. The validity of the solar spectral model, the canopy energy balance, and the ground surface energy balance is only established indirectly by comparing measured and calculated soil temperatures and, to a lesser extent, by comparing measured and calculated snowmelt. A more direct assessment of the solar spectral model and the canopy and ground surface energy balances would include comparisons against measured short- and longwave radiation and measured sensible and latent heat fluxes. Such detailed data are unavailable for the Dry Creek Area. The broadband solar radiation data that are available for the area were used to calculate cloud cover and could therefore not be used to independently verify the calculated incoming solar radiation. Future model applications in areas with more elaborate data sets will be useful to further verify the model calculated energy balances.

### Appendix A: Canopy Energy Balance Equations

The solar energy absorbed by the vegetation has direct beam and diffuse components that are calculated separately for the visible and near-infrared wavebands (after Oleson et al., 2004):

\[
I_{\text{nc}}(\lambda) = I_{\text{dir}}(\lambda) \left\{ 1 - I_{\text{mu}}(\lambda) \right\} \left( 1 - \alpha_{g}(\lambda) \right) \left\{ 1 - \alpha_{c}(\lambda) \right\} \exp\left[ -\text{OD} \left( \text{LAI}_{c} + \text{SAI}_{c} \right) \right] \quad [A1a]
\]

\[
I_{\text{nc}}(\lambda) = I_{\text{dif}}(\lambda) \left\{ 1 - I_{\text{mu}}(\lambda) \right\} \left( 1 - \alpha_{g}(\lambda) \right) \left\{ 1 - \alpha_{c}(\lambda) \right\} \exp\left[ -\text{OD} \right] \quad [A1b]
\]

where \( I^{\text{mu}} \) and \( I^{\text{d}} \) are upward diffuse fluxes away from the vegetation per unit incident direct beam and diffuse flux, respectively, \( I^{\text{mu}} \) and \( I^{\text{d}} \) are downward diffuse fluxes below the vegetation per unit incident direct beam and diffuse radiation, respectively, and OD is the dimensionless optical depth for direct beam radiation. The diffuse fluxes are calculated using canopy radiative transfer relationships developed by Dickinson (1983) and Sellers (1985). The optical depth calculation is based on Sellers (1985). A complete overview of the calculation procedure is given by Oleson et al. (2004). The calculation of the longwave radiation absorbed by the vegetation follows Bonan (1996):

\[
I_{\text{nc}}(\lambda) = \beta_{c} \left( L_{\text{sky}} + L_{g}^{\uparrow} \right) - 2\varepsilon_{c}(T_{c} + 273.15)^{4} \quad [A2]
\]

where \( \beta_{c} \) is the canopy absorptivity, \( \varepsilon_{c} \) is the canopy emissivity, and \( L_{g}^{\uparrow} \) is the upward longwave radiation from the ground:

\[
L_{g}^{\uparrow} = \left( 1 - \beta_{g} \right) L_{g}^{\downarrow} + \varepsilon_{g}(T_{g} + 273.15)^{4} \quad [A3]
\]
where $\beta$, is the ground absorptivity, $\varepsilon_g$ is the ground emissivity, $T_g$ is the ground temperature, and $L_{c,\downarrow}$ is the downward longwave radiation below the canopy:

$$L_{c,\downarrow} = (1 - \beta_g) L_{c,\sky} + \varepsilon_g \sigma (T_g + 273.15)^4$$  \[A4\]

In practice, it is often assumed that absorptivity equals emissivity (Oleson et al., 2004). For this study, we adopted $\varepsilon_g = 0.96$ (soil), $\varepsilon_g = 0.97$ (snow), and

$$\varepsilon_c = 1 - \exp \left( - \frac{(\text{LAI}_c + \text{SAI}_c)}{\bar{z}} \right)$$  \[A5\]

where $\bar{z} = 1$ is the average inverse optical depth for longwave radiation (Bonan, 1996). The sensible heat flux from the vegetation is (Dickinson et al., 1993; Bonan, 1996; Oleson et al., 2004)

$$H_c (T_c) = \rho_a e_c (T_c - T_{ca}) (\text{LAI}_c + \text{SAI}_c) C_{\text{leaf}}$$  \[A6\]

where $\rho_a$ is the density of air, $e_c$ is the specific heat capacity of air, $T_{ca}$ is the canopy-air temperature, and $C_{\text{leaf}}$ is the leaf boundary conductance:

$$C_{\text{leaf}} = 0.01 \frac{\nu_{ca}}{\phi_m}$$  \[A7\]

The factor 0.01 is the turbulent transfer coefficient between the canopy surface and the canopy air (in m s$^{-0.5}$), $d_{\text{leaf}}$ is the characteristic dimension of the leaves in the direction of wind flow, and $\nu_{ca}$ is the estimated wind velocity within the foliage layer:

$$\nu_{ca} = \nu_a \sqrt{\frac{C_{\text{DNv}}}{\varphi_m}}$$  \[A8\]

where $\nu_a$ is the wind velocity at the height above the soil surface at which local meteorological data are being collected, $C_{\text{DNv}}$ is the neutral drag coefficient for momentum, and $\varphi_m$ is a stability-correction factor for momentum (see Dingman, 2002, Appendix D). The canopy-air temperature is a weighted average of the air, canopy, and ground temperatures:

$$T_{ca} = \left( \frac{\nu_a C_{\text{DNv}}}{\varphi_m \varphi_h} \right) T_s + (\text{LAI}_c + \text{SAI}_c) C_{\text{leaf}} T_c + \nu_a C_{\text{gas}} T_g$$  \[A9\]

where $C_{\text{DNv}}$ is the neutral drag coefficient for sensible heat, $\varphi_h$ is a stability-correction factor for sensible heat (see Dingman, 2002), and $C_{\text{gas}}$ is a dimensionless transfer coefficient between the ground and the canopy air that is calculated by weighing the contributions of bare ground and shaded ground (Oleson et al., 2004). The latent heat flux from plant transpiration is calculated as

$$Q_{lw} (T_c) = \begin{cases} (1 - \varepsilon_p) \frac{\gamma_p 0.622}{R_s (T_c + 273.15)} [\varepsilon_p (T_c) - \varepsilon_c] \frac{C_{\text{gas}} C_{\text{min}}}{C_{\text{gas}} + C_{\text{min}}} \varepsilon_c (T_c) > \varepsilon_c & \varepsilon_c (T_c) > \varepsilon_c, \varepsilon_c (T_c) \leq \varepsilon_c & \varepsilon_c (T_c) \leq \varepsilon_c \end{cases}$$  \[A10\]

where $\gamma_p$ is the latent heat of vaporization ($= 2.495 - 2.36 \times 10^{-3} T$ MJ kg$^{-1}$), $R_s$ is the gas constant for air ($= 287$ J kg$^{-1}$ K$^{-1}$), $\varepsilon_p$ is the saturation vapor pressure, $\varepsilon_c$ is the canopy-air vapor pressure, and $C_{\text{stom}}$ is the stomatal conductance. The factor 0.622 is the ratio between the molecular weight of water vapor and the molecular weight of air. The canopy-air vapor pressure is a weighted average of the air, canopy, and ground vapor pressures:

$$\varepsilon_c = \frac{\left( \frac{\varepsilon_p C_{\text{gas}}}{\varphi_p} \right) \varepsilon_p + \text{LAI}_c C_{\text{min}} \varepsilon_c (T_c) + \nu_a C_{\text{gas}} F_s (T_c)}{\left( \frac{\varepsilon_p C_{\text{gas}}}{\varphi_p} \right) \varepsilon_p + \text{LAI}_c C_{\text{min}} \varepsilon_c (T_c) + \nu_a C_{\text{gas}} F_s (T_c)}$$  \[A11\]

where $C_{\text{DNv}}$ is the neutral drag coefficient for latent heat, $\varphi_p$ is a stability-correction factor for latent heat (see Dingman, 2002), $C_{\text{min}}$ is the average conductance of foliage to vapor flux, $\delta$ is equal to either $F_{\text{wat}}$ for $\varepsilon_0 (T_c) \geq \varepsilon_c$ or 1 for $\varepsilon_0 (T_c) < \varepsilon_c$, and $F_s$ is equal to either $\exp[h mg(R(T + 273.15))]$ for 1 (snow) or 1 (snow), with $h$ being the near-surface soil water pressure head, $m$ the molar mass of water (0.018 kg mol$^{-1}$), $g$ the acceleration due to gravity (9.81 m s$^{-2}$), $R$ the gas constant (8.3 J mol$^{-1}$ K$^{-1}$), and $T$ the near-surface soil temperature. The average conductance of foliage is (Dickinson et al., 1993)

$$C_{\text{fol}} = \left( 1 - \frac{1 - F_{\text{wat}}}{F_{\text{wat}}} \right) \frac{C_{\text{stom}}}{C_{\text{leaf}} + C_{\text{stom}}} C_{\text{leaf}} \varepsilon_0 (T_c) > \varepsilon_c \quad \varepsilon_0 (T_c) \leq \varepsilon_c \quad \varepsilon_0 (T_c) \leq \varepsilon_c$$  \[A12\]

$$C_{\text{stom}} = \frac{\overline{\vartheta}(h)}{r_{\text{cmin}} F_l}$$  \[A13\]

where $\overline{\vartheta}$ is the profile-average dimensionless root water uptake reduction function that depends on the soil water pressure head (Feddes et al., 1978), $r_{\text{cmin}}$ is the minimum canopy surface resistance taken as 100 s m$^{-1}$, and $F_l$ gives the dependence on the visible part of the net solar radiation that is absorbed by the canopy (about 1 for overhead sun, and $r_{\text{cmax}}/r_{\text{cmin}}$ at night, with $r_{\text{cmax}}$ being the maximum canopy surface resistance taken as 5000 s m$^{-1}$):

$$F_l = 1 + \left( \frac{I_{nc} (\text{vis}) + I_{nc} (\text{vis})}{I_{ref} + r_{\text{cmin}}/r_{\text{cmax}}} \right)^{I_{nc} (\text{vis}) + I_{nc} (\text{vis})}/I_{ref} + r_{\text{cmin}}/r_{\text{cmax}}$$  \[A14\]

where $I_{\text{ref}}$ is a reference value of the photosynthetically active net solar radiation at the canopy (taken to be 30 W m$^{-2}$). No temperature-dependence factor was included in the calculation of $C_{\text{stom}}$. The latent heat flux from the wet fraction of the canopy $Q_{\text{lw}}$ is

$$Q_{\text{lw}} (T_c) = \delta \frac{\gamma_p 0.622}{R_s (T_s + 273.15)} [\varepsilon_p (T_c) - \varepsilon_c] (\text{LAI}_c + \text{SAI}_c) C_{\text{leaf}}$$  \[A15\]
Appendix B: Surface Energy Balance Equations

The advected heat by rain is

\[ Q_r = \rho_r' \rho_w \varepsilon_a \max(T_a,0) \]  \hspace{1cm} [B1]

where \( \rho_r' \) is the rainfall rate after canopy interception, \( \rho_w \) is the density of water, and \( \varepsilon_a \) is the specific heat of water. The net incoming direct and diffuse solar radiation for a surface that is shaded by vegetation is (after Oleson et al., 2004)

\[ I_{ng}(\lambda) = I_{di}(\lambda) \exp[-\text{OD}(\text{LAI}_e + \text{SAL}_e)]\left[1 - \alpha_g^\mu(\lambda)\right] \hspace{1cm} [B2] \]

\[ + \left[I_{di}(\lambda)I_0^\mu(\lambda) + I_{di}(\lambda)I_0^\mu(\lambda)\right]\left[1 - \alpha_g^\mu(\lambda)\right] \]

where \( I_0^\mu \) and \( I_0^\mu \) are downward diffuse fluxes below the vegetation per unit incident direct beam and diffuse radiation, respectively (Dickinson, 1983; Sellers, 1985), and OD is the dimensionless optical depth for direct beam radiation (Sellers, 1985). The net incoming direct and diffuse solar radiation for a surface that is not shaded by vegetation (exp[-OD(LAI\_e + SAL\_e)] = 1, \( I_0^\mu = 0 \), and \( I_0^\mu = 1 \)) simplifies to

\[ I_{ng}(\lambda) = I_{di}(\lambda)\left[1 - \alpha_g^\mu(\lambda)\right] + I_{di}(\lambda)\left[1 - \alpha_g^\mu(\lambda)\right] \hspace{1cm} [B3] \]

The net incoming longwave radiation for a shaded surface is (Bonan, 1996)

\[ \lambda_{ng} = \beta_g \lambda_{sky} - \varepsilon_g \sigma(T_a + 273.15)^4 \hspace{1cm} [B4] \]

where \( \beta_g \) is the ground absorptivity, \( \varepsilon_g \) is the ground emissivity, and \( \lambda_{sky} \) is the downward longwave radiation below the canopy. Similarly, for an unshaded surface we have

\[ \lambda_{ng} = \beta_g \lambda_{sky} - \varepsilon_g \sigma(T_a + 273.15)^4 \hspace{1cm} [B5] \]

The outgoing sensible heat flux at the surface is

\[ H_g = \frac{\rho_a \varepsilon_a (T_e - T_a) \varphi_m C_{\text{DN}}}{\varphi_m \rho_a \sigma} \hspace{1cm} [B6] \]

where \( \rho_a \) is the density of air, \( \varepsilon_a \) is the specific heat capacity of air, \( T_e \) is the canopy-air temperature, \( \varphi_m \) is the wind velocity at the height above the soil surface at which local meteorological data are being collected, \( \varepsilon_a \) is the estimated wind velocity within the foliage layer, \( C_{\text{DN}} \) is a dimensionless transfer coefficient between the ground and the canopy that air is calculated by weighting the contributions of bare ground and shaded ground (Oleson et al., 2004), \( C_{\text{DN}} \) is the neutral drag coefficient for sensible heat, and \( \varphi_m \) and \( \varphi_r \) are stability-correction factors for momentum and sensible heat, respectively (see Dingman, 2002, Appendix D). The outgoing latent heat flux for a snow-covered surface is calculated as

\[ Q_e = \frac{\gamma_0 \cdot 0.622}{R_e(T_a + 273.15)} \left[T_e - T_a\right] \varphi_m C_{\text{DN}} \hspace{1cm} [B7] \]

\[ \frac{\gamma_0 \cdot 0.622}{R_e(T_a + 273.15) \varphi_m \varphi_r} \hspace{1cm} \text{unvegetated} \]

where \( \gamma_0 \) is the latent heat of sublimation (2834 J g\(^{-1}\)), \( R_e \) is the gas constant for air (= 287 J kg\(^{-1}\) K\(^{-1}\)), \( \varepsilon_0 \) is the saturation vapor pressure, \( \varepsilon_a \) is the canopy-air vapor pressure, \( C_{\text{DN}} \) is the neutral drag coefficient for latent heat, and \( \varphi_r \) is a stability-correction factor for latent heat (see Dingman, 2002). The factor 0.622 is the ratio between the molecular weight of water vapor and the molecular weight of air. The outgoing latent heat flux for soil surfaces without snow is

\[ Q_e = \frac{\gamma_0 \cdot 0.622}{R_e(T_a + 273.15)} \left[T_e - T_a\right] \varphi_m C_{\text{DN}} \hspace{1cm} \text{vegetated} \]

\[ \frac{\gamma_0 \cdot 0.622}{R_e(T_a + 273.15) \varphi_m \varphi_r} \hspace{1cm} \text{unvegetated} \]

where \( \gamma_0 \) is the latent heat of vaporization (= 2.495 - 2.36 \times 10^{-3} T \text{ MJ kg}^{-1}\), \( F_i \) is the vapor pressure reduction factor (see Appendix A), and \( F_{lea} T_a < \varepsilon_a \) and \( F_{lea} T_a < \varepsilon_a \) are the conditions for dew formation for vegetated and unvegetated soil surfaces, respectively.

Appendix C: Notation

- root water uptake reduction function
- coefficients in total solar radiation calculation (subscripts 1 and 2), variable units
- conductance (subscripts fol, leaf, and stom), m s\(^{-1}\)
- transfer coefficient (subscript gca)
- neutral drag coefficient (subscripts h, m, and v)
- volumetric heat capacity (subscripts a, i, old, r, so, and w), J m\(^{-3}\) K\(^{-1}\)
- snow compaction rate, s\(^{-1}\)
- cloud cover
- constants in snow compaction calculation (subscripts 1, 2, 3, 4, 5, and 6), variable units
- specific heat capacity (subscripts a and w), J kg\(^{-1}\) K\(^{-1}\)
- effective diffusion coefficient for water vapor in snow, m\(^2\) s\(^{-1}\)
- snow or soil layer thickness, m
- correction factor for the Earth–sun distance
- snow grain diameter, m
- characteristic leaf dimension in the direction of wind flow, m
- vapor pressure (subscripts 0, a, and ca), Pa
- fraction through the step
- factor of aerosol scatter that is directed downward
- mass of water that is retained per mass of dry snow
- factor in the stomatal conductance calculation
- Kemper number
- fraction in canopy-air vapor pressure calculation
- canopy wetted fraction
- acceleration due to gravity, m s\(^{-2}\)
- sensible heat flux (subscripts c and g), J m\(^{-2}\) s\(^{-1}\)
- soil water pressure head, m
- surface water ponding height, m
- bubbling pressure head, m
- equilibrium potential of liquid water in contact with ice, m
- osmotic head due to dissolved ions, m